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On the Solvability of an Inverse Problem for the Kinetic Equation

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Abstract: In this study, the solvability conditions of an inverse problem for the stationary kinetic equation is investigated. Also, a symbolic algorithm based on the Galerkin method is developed for computing the approximate solution of the problem.
Keywords: Kinetic equation, inverse problem, symbolic computation

Kinetik Denklem için bir Ters Problemin Çözülebilirliği Üzerine

Özet: Bu çalışmada, durağan kinetik denklem için bir ters problemin çözülebilirlik şartları araştırılmıştır. Ayrıca, bu problemin yaklaşık çözümünü hesaplamak için Galerkin metoduna dayanan bir sembolik algoritma geliştirilmiştir.
Anahtar Kelimeler : Kinetik denklem, ters problem, sembolik hesaplama

1. Introduction

Kinetic equations (KE) are widely used for qualitative and quantitative description of physical, chemical, biological, and other kinds of processes on a microscopic scale. They are often referred to as master equations since they play an important role in the theory of substance motion under the action of forces, in particular, irreversible processes, [1,2].

An inverse problem for KE is a problem of simultaneous determining the distribution function of a quantity and some functions entering the equations for given additional information. As a rule, the additional information is the trace of the distribution function on some manifolds of variables. Inverse problems for KE are important both from theoretical and practical points of view. The physical interpretation of these problems consists in finding particle interaction forces, scattering indicatrices, radiation sources and other physical parameters. Interesting results in this field are presented in [3-10].

In this paper, the existence, uniqueness and stability of the solution of an inverse problem for the stationary kinetic equation is proven in the case where the values of the solution are known on the boundary of a domain. A symbolic computation approach based on the Galerkin method is developed to obtain the approximate solution of the problem. A comparison between the computed approximate solution and the exact solution of the problem is presented.

We consider the kinetic equation

$$\sum_{i=1}^{n}\left(v_i \frac{\partial u(x,v)}{\partial x_i} + f_i \frac{\partial u(x,v)}{\partial v_i} \right) = \sigma(x), \tag{1}$$

in the domain $\Omega = \{(x,v): x \in D \subset \mathbb{R}^n, v \in G \subset \mathbb{R}^n, n \geq 1\}$ where $\partial D, \partial G \in C^3$, $\partial\Omega = \overline{\Gamma}_1 \cup \overline{\Gamma}_2$, $\Gamma_1 = \partial D \times G$, $\Gamma_2 = D \times \partial G$ and $\overline{\Gamma}_1$, $\overline{\Gamma}_2$ are the closures of Γ_1, Γ_2 respectively.

Equation 1 is extensively used in plasma physics and astrophysics, [1,2]. In applications, u represents the number (or the mass) of particles in the unit volume element of the phase space in the neighbourhood of the point (x,v), and $f = (f_1, f_2, ..., f_n)$ is the force acting on a particle.

2. Formulation of the Problem

Problem 1. *Determine the functions $u(x,v)$ and $\sigma(x)$ defined in Ω from equation (1), provided that the function f is given and the trace of u is known on the boundary.*

The main difficulty in studying the solvability of problem 1 is overdeterminacy. In the theory of inverse problems, usually "overdeterminacy" means that the number of free variables in the data exceeds the number of free variables in the unknown coefficient or right hand side of the equation ($\sigma(x)$), and this is not the case for $n=1$ here, whereas for dimension $n \geq 2$ Problem 1 is overdetermined in the last sense. It is important to note here that inverse problems for KE and integral geometry problems are closely interrelated. And the underlying operator of the related IGP is compact and its inverse operator is unbounded. Therefore, it is impossible to prove general existence results. This is the true reason for why we use the term "overdeterminacy" in this sense here.

In the paper, using some extension of the class of unknown functions, the overdetermined inverse problem is replaced by a related determined one, which is a new and interesting technique of investigating the solvability of overdetermined problems. This method was firstly proposed by Amirov (1986) for the transport equation.

Problem 2. *Find a pair of functions* (u,σ) *defined in* Ω. *that satisfies the relations:*

$$Lu = \sigma(x,v), \tag{2}$$

$$u\big|_{\partial\Omega} = u_0 , \tag{3}$$

$$\hat{L}\sigma = 0, \; \hat{L} = \sum_{i=1}^{n} \frac{\partial^2}{\partial x_i \partial v_i}, \tag{4}$$

provided that the function f *is given.*

Here equation (4) is satisfied in generalized functions sense, i.e., $\left\langle \sigma, \hat{L}^* \eta \right\rangle = 0$ for any $\eta \in C_0^\infty(\Omega)$.

3. Solvability of the Problem

To formulate the solvability theorem for Problem 2, we need the following notation:

$\Gamma(A)$ denotes the set of functions $u(x,v)$ with the following properties

i) For $u \in \Gamma(A)$, $Au \in L_2(\Omega)$ in the generalized sense, where $Au = \hat{L}Lu$;

ii) There exists a sequence $\{u_k\} \subset \widetilde{C}_0^3 \quad \{\varphi : \varphi \in C^3(\Omega), \varphi|_{\partial\Omega} = 0\}$ such that $u_k \to u$ in $L_2(\Omega)$ and $\langle Au_k, u_k \rangle \to \langle Au, u \rangle$ as $k \to \infty$.

The condition $Au \in L_2(\Omega)$ in the generalized sense means that there exists a function $\Im \in L_2(\Omega)$ such that for all $\varphi \in C_0^\infty(\Omega)$, $\langle u, A^*\varphi \rangle = \langle \Im, \varphi \rangle$ and $Au = \Im$, where A^* is the differential operator conjugate to A in the sense of Lagrange.

The standard spaces $C^m(\Omega)$, $L_2(\Omega)$ and $H^k(\Omega)$ are described in detail, for example, in [11,12].

Theorem 1. *Suppose that* $f \in C^1(\Omega)$ *and the inequality*

$$\sum_{i,j=1}^{n} \frac{\partial f_i}{\partial x_j} \xi^i \xi^j \geq \alpha_1 |\xi|^2 \tag{5}$$

holds for all $\xi \in \mathbb{R}^n$, *where* α_1 *is a positive number. Then Problem* 2 *has at most one solution* (u, σ) *such that* $u \in \Gamma(A)$ *and* $\sigma \in L_2(\Omega)$.

Proof. Let (u, σ) be a solution to Problem 2 such that $u = 0$ on $\partial\Omega$. and $u \in \Gamma(A)$. Equation 1 and condition (4) imply $Au = 0$. Since $u \in \Gamma(A)$, there exists a sequence $\{u_k\} \subset \widetilde{C}_0^3$ such that $u_k \to u$ in $L_2(\Omega)$ and $\langle Au_k, u_k \rangle \to 0$ as $k \to \infty$. Observing that $u_k = 0$ on $\partial\Omega$, we get

$$-\langle Au_k, u_k \rangle = \sum_{i=1}^{n} \left\langle \frac{\partial}{\partial v_i}(Lu_k), u_{k_{x_i}} \right\rangle. \tag{6}$$

The right-hand side of (6) can be estimated as follows:

$$2\sum_{i=1}^{n} \frac{\partial u_k}{\partial x_i} \frac{\partial}{\partial v_i}(Lu_k) = \sum_{i=1}^{n} \left(\frac{\partial u_k}{\partial x_i}\right)^2 + \sum_{i,j=1}^{n} \frac{\partial f_i}{\partial x_j} \frac{\partial u_k}{\partial v_i} \frac{\partial u_k}{\partial v_j} + \sum_{i,j=1}^{n} \frac{\partial}{\partial v_j}\left(v_i \frac{\partial u_k}{\partial x_i} \frac{\partial u_k}{\partial x_j}\right)$$

$$+\sum_{i,j=1}^{n} \frac{\partial}{\partial x_i}\left(v_i \frac{\partial u_k}{\partial v_j} \frac{\partial u_k}{\partial x_j}\right) - \sum_{i,j=1}^{n} \frac{\partial}{\partial x_j}\left(v_i \frac{\partial u_k}{\partial x_i} \frac{\partial u_k}{\partial v_j}\right) - \sum_{i,j=1}^{n} \frac{\partial}{\partial x_j}\left(f_i \frac{\partial u_k}{\partial v_i} \frac{\partial u_k}{\partial v_j}\right)$$

$$+\sum_{i=1}^{n} \frac{\partial}{\partial v_i}\left(v_i \left(\frac{\partial u_k}{\partial x_i}\right)^2\right) + \sum_{i,j=1}^{n} \frac{\partial}{\partial v_j}\left(f_i \frac{\partial u_k}{\partial v_i} \frac{\partial u_k}{\partial x_j}\right) + \sum_{i,j=1}^{n} \frac{\partial}{\partial v_i}\left(f_i \frac{\partial u_k}{\partial v_j} \frac{\partial u_k}{\partial x_j}\right). \tag{7}$$

Taking into account the geometry of Ω and the condition $u_k = 0$ on $\partial\Omega$, from (7)

$$-\langle Au_k, u_k \rangle = J(u_k) \tag{8}$$

is obtained, where

$$J(u_k) \equiv \frac{1}{2}\sum_{i=1}^{n}\int_{\Omega}\left(\left(\frac{\partial u_k}{\partial x_i}\right)^2 + \sum_{j=1}^{n}\frac{\partial f_i}{\partial x_j}\frac{\partial u_k}{\partial v_i}\frac{\partial u_k}{\partial v_j}\right)d\Omega. \tag{9}$$

Since Ω is bounded and $u_k\big|_{\partial\Omega} = 0$, from (9) and Steklov inequality it follows that

$$J(u_k) > \frac{1}{2}\int_{\Omega}\left|\nabla_x u_k\right|^2 d\Omega \geq c\int_{\Omega}\left|u_k\right|^2 d\Omega, \tag{10}$$

where $c > 0$, $\nabla_x u_k = \left(u_{k_{x_1}}, u_{k_{x_2}}, ..., u_{k_{x_n}}\right)$. Using definition of $\Gamma(A)$, we have $\int_{\Omega}|u|^2 d\Omega \leq 0$ and $u = 0$ in Ω. Then (2) implies $\sigma(x,v) = 0$. Hence uniqueness of the solution of the problem is proven.

Since $u_0 \in C^3(\partial\Omega)$, $\partial D \in C^3$, $\partial G \in C^3$ then from Theorem 2, Sec. 4.2., Chapter III in [12], Problem 2 can be reduced to the following problem.

Problem 3. *Determine the pair* (u,σ) *from the equation*

$$Lu = \sigma(x,v) + F$$

provided that $F \in H^2(\Omega)$ *is given, the trace of the solution* u *on the boundary* $\partial\Omega$ *is zero and* σ *satisfies equation (4).*

Theorem 2. *Under the assumptions of Theorem 1, suppose that* $F \in H^2(\Omega)$. *Then there exists a solution* (u,σ) *of Problem* 3 *such that* $u \in \Gamma(A) \cap H^1(\Omega)$, $\sigma \in L_2(\Omega)$.

Proof. We consider the following auxiliary problem

$$Au = \Im, \tag{11}$$

$$u\big|_{\partial\Omega} = 0, \tag{12}$$

where $\Im = \hat{L}F$. We select a set $\{w_1, w_2, ...\} \subset \tilde{C}_0^3$, which is a complete and orthonormal set in $L_2(\Omega)$. We may assume here that the linear span of this set is everywhere dense in $H_{1,2}(\Omega)$. $H_{1,2}(\Omega)$ is the set of all real-valued functions $u(x,v) \in L_2(\Omega)$ that have generalized derivatives $u_{x_i}, u_{v_i}, u_{x_i v_j}, u_{v_i v_j}, (i, j = 1, ...n)$, belong to $L_2(\Omega)$ and whose trace on $\partial\Omega$ is zero.

For problem (11)-(12), an approximate solution

$$u_N = \sum_{i=1}^{N} \alpha_{N_i} w_i \,, \ \alpha_N = (\alpha_{N_1}, \alpha_{N_2}, ..., \alpha_{N_N}) \in \mathbb{R}^n \,, \tag{13}$$

is defined as a solution to the following problem:

Find the vector α_N from the system of linear algebraic equations

$$\left\langle Au_N - \Im, w_i \right\rangle = 0, \ i = 1, 2, ..., N \,. \tag{14}$$

We shall prove that under the hypotheses of Theorem 2, system (14) has a unique solution α_N for any function $F \in H^2(\Omega)$. For this purpose, i th equation of the homogeneous system ($\Im = 0$) is multiplied by $-2\alpha_{N_i}$ and sum from 1 to N with respect to i. Hence $-2\langle Au_N, u_N \rangle = 0$ is obtained. From (5) and (8), we obtain $\nabla u_N = 0$, where $\nabla u_N = \left(u_{N_{x_1}}, ..., u_{N_{x_n}}, u_{N_{v_1}}, ..., u_{N_{v_n}} \right)$.

So, $u_N = 0$ in Ω as a result of the conditions $u_N = 0$ on $\partial\Omega$, $u_N \in \widetilde{C}_0^3(\Omega)$. Since the system $\{w_i\}$ is linearly independent, we get $\alpha_{N_i} = 0$, $i = 1, 2, ..., N$. Thus the homogeneous version of system (14) has only a trivial solution and therefore the original inhomogeneous system (14) has a unique solution $\alpha_N = \left(\alpha_{N_i} \right)$, $i = 1, 2, ..., N$ for any function $F \in H^2(\Omega)$.

Now we estimate u_N, in terms of F. We multiply the i th equation of the system by $-2\alpha_{N_i}$ and sum from 1 to N with respect to i. Since $\Im = \hat{L}F$, we obtain

$$-2\langle Au_N, u_N \rangle \quad -2\left\langle \hat{L}F, u_N \right\rangle . \tag{15}$$

Observing that $u_N = 0$ on $\partial\Omega$, the right-hand side of (15) can be estimated as follows

$$-2\left\langle \hat{L}F, u_N \right\rangle \quad 2\int_\Omega \sum_{i=1}^{n} \frac{\partial F}{\partial v_i} \frac{\partial u_N}{\partial x_i} d\Omega \le \beta \int_\Omega \left| \nabla_v F \right|^2 d\Omega + \beta^{-1} \int_\Omega \left| \nabla_x y_N \right|^2 d\Omega \,, \tag{16}$$

where $\beta^{-1} < \alpha_1$, $\nabla_v F = \left(F_{v_1}, ..., F_{v_n} \right)$. From (8), we have

$$2J(u_N) \le \beta \int_\Omega \left| \nabla_v F \right|^2 d\Omega + \beta^{-1} \int_\Omega \left| \nabla_x y_N \right|^2 d\Omega \,, \tag{17}$$

and since Ω is bounded and $u_N = 0$ on $\partial\Omega$, from (17), we have

$$\| u_N \|_{\overset{\circ}{H}{}^1(\Omega)} \le C \| \nabla_v F \|_{L_2(\Omega)} \,, \tag{18}$$

where the constant $C > 0$ does not depend on N.

Thus, the set of functions u_N, $N = 1, 2, \ldots$, is bounded in $\overset{\circ}{H}{}^1(\Omega)$. Since $\overset{\circ}{H}{}^1(\Omega)$ is a Hilbert space, there exists a subsequence in this set that is denoted again by $\{u_N\}$ converging weakly in $\overset{\circ}{H}{}^1(\Omega)$ to a certain function $u \in \overset{\circ}{H}{}^1(\Omega)$. From inequality (18) and weak convergence of $\{u_N\}$ to $u \in \overset{\circ}{H}{}^1(\Omega)$, it follows that

$$\|u\|_{\overset{\circ}{H}{}^1(\Omega)} \le \lim_{N\to\infty} \|u\|_{\overset{\circ}{H}{}^1(\Omega)} \le C \left\|\nabla_v F\right\|_{L_2(\Omega)}. \quad (19)$$

From estimate (18), it is easy to prove that there exists a subsequence of $\{u_N\}$ and

$$\left\langle Lu_N - F, \hat{L}\eta \right\rangle = 0. \quad (20)$$

Since the linear span of the functions w_i, $i = 1, 2, \ldots$, is everywhere dense in $\overset{\circ}{H}_{1,2}(\Omega)$, passing to the limit as $N \to \infty$ in (20), yields to

$$\left\langle Lu_N - F, \hat{L}\eta \right\rangle = 0 \quad (21)$$

for any $\eta \in \overset{\circ}{H}_{1,2}(\Omega)$. If we set $\sigma = Lu - F$, from (21) we see that the function σ satisfies the condition (4) and from (18) the following estimate is valid:

$$\|\sigma\|_{L_2(\Omega)} \le C \left\|\nabla_v F\right\|_{L_2(\Omega)} + \|F\|_{L_2(\Omega)} \quad (22)$$

Thus we have found a solution (u, σ) to Problem 3, where $u \in \overset{\circ}{H}{}^1(\Omega)$, $\sigma \in L_2(\Omega)$. Now we will show that $u \in \Gamma(A)$. Since $u \in L_2(\Omega)$ and $F \in H^2(\Omega)$, it follows that $\Im \quad Au \in L_2(\Omega)$ in the generalized sense. Indeed, for any $\eta \in C_0^\infty(\Omega) \subset \overset{o}{H}_{1,2}(\Omega)$, the following equalities hold:

$$\left\langle u, A^*\eta \right\rangle = \left\langle u, L^*\hat{L}\eta \right\rangle \quad \left\langle Lu, \hat{L}\eta \right\rangle \quad \left\langle F, \hat{L}\eta \right\rangle \quad \left\langle \Im, \eta \right\rangle. \quad (23)$$

Now, we have to show that $\langle Au_N, u_N \rangle \to \langle Au, u \rangle$ as $N \to \infty$. Let's denote the orthogonal projector of $L_2(\Omega)$ onto M_n by P_n, where M_n is the linear span of the set $\{w_1, w_2, \ldots, w_n\}$. We have $P_N A u_N = P_N \Im$ from (14) and $P_N \Im$ strongly converges to $\Im$ in $L_2(\Omega)$ as $N \to \infty$. Then we have $\langle P_N A u_N, u_N \rangle \to \langle Au, u \rangle$ as $N \to \infty$ because $\{u_N\}$

weakly converges to u and $\{P_N Au_N\}$ strongly converges to Au in $L_2(\Omega)$ as $N\to\infty$. Since P_N is self adjoint in $L_2(\Omega)$, we obtain

$$\langle Au_N, u_N\rangle = \langle Au_N, P_N u_N\rangle = \langle P_N Au_N, u_N\rangle \qquad (24)$$

Consequently, we obtain the convergence $\langle Au_N, u_N\rangle \to \langle Au, u\rangle$ as $N\to\infty$, which completes the proof.

4. Solution Algorithm and Some Computational Experiments

An approximate solution to Problem 3 will be sought in the form

$$u_N = \sum_{i_1,i_2,\dots,i_n,j_1,j_2,\dots,j_n=0}^{N-1} \alpha_{N_{i_1,i_2,\dots,i_n,j_1,j_2,\dots,j_n}} w_{i_1,i_2,\dots,i_n,j_1,j_2,\dots,j_n}\,\eta(x)\,\mu(v) \qquad (25)$$

for the domains, for example, $D=\{x:|x|<1\}\subset\mathbb{R}^n$, $G=\{v:|v|<1\}\subset\mathbb{R}^n$,

where $w_{i_1,i_2,\dots,i_n,j_1,j_2,\dots,j_n} = x_1^{i_1}x_2^{i_2}\dots x_n^{i_n}v_1^{j_1}v_2^{j_2}\dots v_n^{j_n}$ and the systems $\{x_1^{i_1}x_2^{i_2}\dots x_n^{i_n}\}_{i_1,\dots,i_n=0}^{\infty}$, $\{v_1^{j_1}v_2^{j_2}\dots v_n^{j_n}\}_{j_1,\dots,j_n=0}^{\infty}$ are complete in $L_2(D)$ and $L_2(G)$, respectively. The functions $\eta(x)$ and $\mu(v)$ are defined as follows

$$\eta(x)=\begin{cases}1-|x|^2, & |x|<1\\ 0, & |x|\ge 1\end{cases}, \qquad \mu(v)=\begin{cases}1-|v|^2, & |v|<1\\ 0, & |v|\ge 1\end{cases},$$

In expression (25), unknown coefficients $\alpha_{N_{i_1,i_2,\dots,i_n,j_1,j_2,\dots,j_n}}$, $i_1,i_2,\dots,i_n, j_1, j_2,\dots, j_n = 0,\dots,N-1$ are determined from the following system of linear algebraic equations (SLAE):

$$\sum_{i_1,i_2,\dots,i_n,j_1,j_2,\dots,j_n=0}^{N-1}\left(A\left(\alpha_{N_{i_1,i_2,\dots,i_n,j_1,j_2,\dots,j_n}} w_{i_1,i_2,\dots,i_n,j_1,j_2,\dots,j_n}\right)\eta\mu, w_{i_1',i_2',\dots,i_n',j_1',j_2',\dots,j_n'}\eta\mu\right)_{L_2(\Omega)}$$

$$=(F, w_{i_1',i_2',\dots,i_n',j_1',j_2',\dots,j_n'})_{L_2(\Omega)} \qquad (26)$$

Algorithm 1.

Input: $N, F(x,v), f(x,v)$

Output: $u_N(x,v), \sigma(x,v)$

{The following procedure computes left side of each equation in (26)}

Procedure *LeftSLAE* $(i_1', i_2',\dots,i_n', j_1', j_2',\dots, j_n')$

Left :=0

For $i_1 = 0,..., N-1$ do, for $i_2 = 0,..., N-1$ do ,..., for $i_n = 0,..., N-1$ do

For $j_1 = 0,..., N-1$ do, for $j_2 = 0,..., N-1$ do ,..., for $j_n = 0,..., N-1$ do

begin

$$Left := Left + \left(A\left(\alpha_{N_{i_1,i_2,...,i_n,j_1,j_2,...,j_n}} w_{i_1,i_2,...,i_n,j_1,j_2,...,j_n}\right)\eta(x)\mu(v), w_{i_1',i_2',...,i_n',j_1',j_2',...,j_n'}\eta(x)\mu(v)\right)_{L_2(\Omega)}$$

end;

{The following procedure constructs system (26)}

Procedure $SLAE$

$Set := \{\ \}$, $\Im = \hat{L}F$

For $i_1' = 0,..., N-1$ do, for $i_2' = 0,..., N-1$ do,…, for $i_n' = 0,..., N-1$ do

For $j_1' = 0,..., N-1$ do, for $j_2' = 0,..., N-1$ do,…, for $j_n' = 0,..., N-1$ do

Begin

$$Set = Set \cup \left\{ LeftSLAE\left(i_1', i_2',..., i_n', j_1', j_2',..., j_n'\right) \quad \left(\Im, w_{i_1',i_2',...,i_n',j_1',j_2',...,j_n'}\eta(x)\mu(v)\right)_{L_2(\Omega)} \right\}$$

end; {Principle part}

Solve $\left(SLAE, \left\{\alpha_{N_{i_1,i_2,...,i_n,j_1,j_2,...,j_n}}\right\}\right)$

For $i_1 = 0,..., N-1$ do, for $i_2 = 0,..., N-1$ do,..., for $i_n = 0,..., N-1$ do

For $j_1 = 0,..., N-1$ do, for $j_2 = 0,..., N-1$ do,..., for $j_n = 0,..., N-1$ do

Begin $u_N = u_N + \left(\alpha_{N_{i_1,i_2,...,i_n,j_1,j_2,...,j_n}} w_{i_1,i_2,...,i_n,j_1,j_2,...,j_n}\right)\eta(x)\mu(v)$ end

$\sigma(x,v) = L(u_N) - F(x,v)$

end of the algorithm.

The algorithm has been implemented in the computer algebra system Maple and tested for several inverse problems. Two examples are presented below. In the examples, U_N shows the computed solution at N, and N is the approximation level in (26).

Example 1. *Let us consider Problem 3 on the domain* $\Omega \quad \left\{(x,v) \mid x \in (-1,1), v \in (2,3)\right\}$, *with the given functions* $F(x,v) = (10-3v)x^4 v + (-15v + 15v^2 - 3v^3 - 10)x^2 v$, $f(x,v) = x$. *Then, at* $N = 2$ *the algorithm gives the result:*

$U_2 = (x - x^3)(6v - 5v^2 + v^3)$, $\sigma_2 = 6v^2 \text{-} 5v^3 + v^4 + 6x^2 - 6x^4$ *and this is also the exact solution of the problem.*

Example 2. *Consider Problem 3 on* $\Omega = \{(x,v) | x \in (-1,1), v \in (-1,1)\}$, *then according to the given functions* $F(x,v) = (-v + v^3)(3x^2 v(v+2) + 6x - 2xe^{2v}(v_1 + 2))$ *and* $f(x,v) = 0$, *computed approximate solution and exact solution* $u(x,v)$ *of the problem at* $N = 2$ *and* $N = 4$ *is presented on Figure 1:(a),(b), respectively. Here, the exact solution of the problem is*

$$u(x,v) = (1 - x^2)(1 - v^2)\left(xv + \frac{3}{v+2} - e^{2v}\right), \ \sigma(x,v) = \frac{(-v + v^3)(6x - v^2 - 2v)}{v+2}.$$

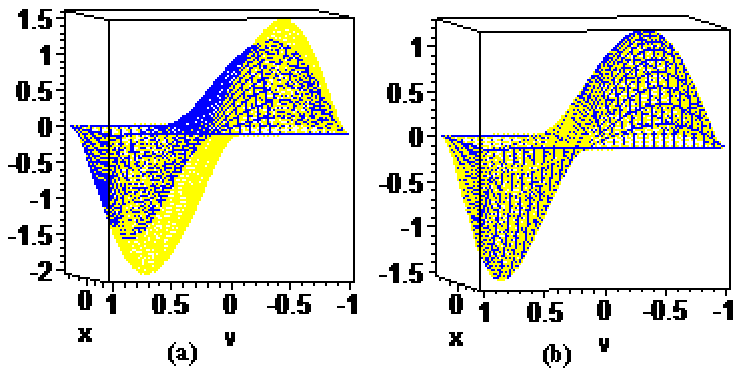

Figure 1. A comparison of the approximate (dotted, yellow graph) and exact solution (solid, blue graph) u of the problem (a) $N = 2$, (b) $N = 4$.

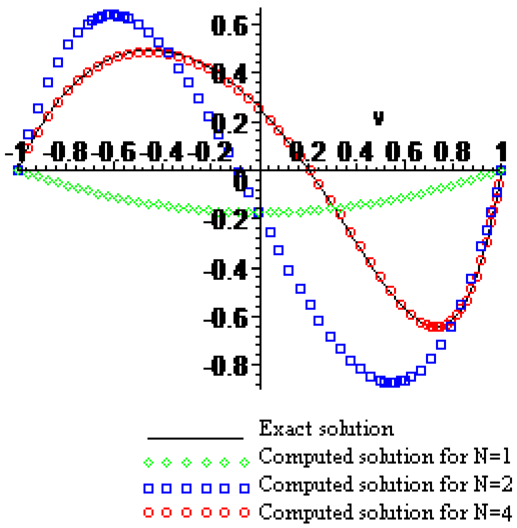

Figure 2. 1-d cross-section comparison of approximate and exact solution (cross) u at $x = 0.7$ for different approximation levels.

In example 1, computed approximate solution at $N = 2$ coincides with the exact solution of the problem and in example 2, as it can be seen from Figure 1-(b) and Figure 2 computed solution at $N = 4$ is very closed to the exact solution. Consequently, the computational experiments show that the proposed algorithm gives efficient and reliable results.

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