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Jordan Left Derivations On Completely PrimeGamma Rings

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Abstract: We define a Jordan left derivation on Γ-rings and show that the existence of a nonzero Jordan left derivation on a completely prime Γ-ring implies Γ-ring is commutative with an assumption. Furthermore, with the same assumption, we show that a Jordan left derivation on completely prime Γ-rings is a left derivation.
Key Words: Gamma ring, completely prime gamma ring, left derivation ,Jordan left derivation.

Özet: Bu çalışmada Γ halkaları üzerinde Jordan sol türev tanımı verilmiş ve tamamen (completely) asal bir Γ halkası üzerinde bir Jordan sol türev varsa, Γ halkasının belirli bir koşulu sağlaması halinde değişme özelliğine sahip olduğu gösterilmiştir. Ayrıca, aynı koşulun sağlanması durumunda tamamen asal bir Γ halkası üzerinde bir Jordan sol türevin bir sol türev olduğu kanıtlanmıştır.
Anahtar Kelimeler: Gamma halka, tamamen asal gamma halka, sol türev, jordan sol türev.

1. Preliminaries

In [2], Bresar and Vukman proved a Jordan derivation on prime rings is a derivation. In [6], Sapancı and Nakajima defined a derivation and a Jordan derivation on Γ-rings and showed that a Jordan derivation on a certain type of completely prime Γ-rings is a derivation. Furthermore, in [3], Bresar and Vukman showed that the existence of a nonzero Jordan left derivation of R into X implies R is commutative where R is a ring and X is a 2-torsion free and 3-torsion free left R-module. In [4] , Jun and Kim proved that their results held without the property 3-torsion free. In this paper, we define a Jordan left derivation on Γ-rings and we show that the existence of a nonzero Jordan left derivation on a 2-torsion free Γ-ring which satisfies the condition $xa\,y\,b\,z = x\,b\,y\,a\,z$ for all $x, y, z \in M$ and $a, b \in \Gamma$ implies Γ-ring is commutative. Also, in the same condition, a Jordan left derivation on a completely prime Γ-ring is a left derivation on Γ-ring.

Let M and Γ be additive abelian groups. M is called a Γ-ring if for any $x, y, z \in M$ and $a, b \in \Gamma$, the following conditions are satisfied:

(1) $x a y \in M$

(2) $(x+y) a z = x a z + y a z$

$x(a + b) z = x a z + x b z$

$x a (y+z) = x a y + x a z$

(3) $(x a y) b z = x a (y b z)$

The notion of a Γ-ring was introduced by Nobusawa [5] and generalized by Barnes [1] as defined above. Many properties of Γ-rings were obtained by many researchers.

Let A, B be subsets of a Γ-ring M and Λ a subset of Γ We denote $A \Lambda B$ the subset of M consisting of all finite sums of the form $\sum a_i l_i b_i$ where $a_i \in A$, $b_i \in B$ and $l_i \in \Lambda$ A right ideal (resp. left ideal) of a Γ-ring M is an additive subgroup I of M such that $I \Gamma M \subset I$ (resp. $M \Gamma I \subset I$). If I is a right and a left ideal in M, then we say that I is an ideal. M is called 2-torsion free if $2a=0$ implies $a=0$ for all $a \in M$.

A Γ-ring M is called prime if $a \Gamma M \Gamma b = 0$ implies $a=0$ or $b=0$ and M is called completely prime if $a \Gamma b = 0$ implies $a=0$ or $b=0$ $(a,b \in M)$. Since $a \Gamma b \Gamma a \Gamma b \subset a \Gamma M \Gamma b$, every completely prime Γ-ring is prime.

Let M be a Γ-ring and let $D: M \to M$ be an additive map. D is called a derivation if for any $a, b \in M$ and $a \in \Gamma$,

$D(a a b) = D(a) a b + a a D(b),$

D is called a left derivation if for any $a, b \in M$ and $a \in \Gamma$,

$D(a a b) = a a D(b) + b a D(a)$,

D is called a Jordan derivation if for any $a \in M$ and $a \in \Gamma$,

$D(a a a) = D(a) a a + a a D(a),$

and D is called a Jordan left derivation if for any $a \in M$ and $a \in \Gamma$

$D(a\alpha a)=2a\alpha D(a)$.

In [6] ,they gave an example of a derivation, a Jordan derivation on a Γ-ring. We added it a Jordan left derivation on same Γ-ring in the following example.

Example 1.1 Let R is a ring, $M = M_{1,2}(R)$ and $\Gamma = \left\{ \begin{bmatrix} n.1 \\ 0 \end{bmatrix} : n \text{ is an integer} \right\}$.Then M is a $\Gamma -$ring. If $d:R \to R$ is a Jordan left derivation and $N=\{(a,a):a \in R\}$ is the subset of M, then N is a Γ-ring and the map $D:N \to N$ defined by $D((a,a))=(d(a),d(a))$ is a Jordan left derivation on N.

2. Left Jordan Derivations

Some parts of the following Lemmas are essentially proved in [2,3,4,6] .

Lemma 2.1 Let M is an arbitrary $\Gamma -$ring and D is a Jordan left derivation on M. Then, for all $a,b \in M$ and for all $\alpha \in \Gamma$:

(i) $D(a\alpha b+b\alpha a)=2a\alpha D(b)+2b\alpha D(a)$.

Especially if M is 2-torsion free and $a\alpha b\beta c = a\beta b\alpha c$ for all $a,b,c \in M$ and $\alpha, \beta \in \Gamma$, then

(ii) $D(a\alpha b\beta a)=a\beta a\alpha D(b)+3a\alpha b\beta D(a)-b\alpha a\beta D(a)$.

(iii) $D(a\alpha b\beta c+c\alpha b\beta a)=a\beta c\alpha D(b)+c\beta a\alpha D(b)+3a\alpha b\beta D(c)+3c\alpha b\beta D(a)$
$-b\alpha c\beta D(a)-b\alpha a\beta D(c)$.

Proof. (i) is obtained by computing $D((a+b)\alpha (a+b))$.

(ii) From (i), $D(a\beta b+b\beta a)=2a\beta D(b)+2b\beta D(a)$. Then replacing $a\alpha b+b\alpha a$ for b, we have

$D(a\alpha b\beta a+a\beta b\alpha a)=2a\beta a\alpha D(b)+4a\beta b\alpha D(a)+2a\alpha b\beta D(a)-2b\alpha a\beta D(a)$.

Then, since $a\alpha b\beta c = a\beta b\alpha c$ for all $a,b,c \in M$ and $\alpha, \beta \in \Gamma$ and M is 2-torsion free we obtain (ii).

(iii)is obtained replacing $a+c$ for a in (ii).

Lemma 2.2 Let M is a 2-torsion free Γ-ring, D is a Jordan left derivation on M and $a\alpha b\beta c = a\beta b\alpha c$ for all $a,b,c \in M$ and $\alpha, \beta \in \Gamma$ then

(i) $(a\alpha b-b\alpha a)\beta a\alpha D(a)=a\alpha (a\alpha b-b\alpha a)\beta D(a)$.

(ii) $(a\alpha b-b\alpha a)\beta (D(a\alpha b)-a\alpha D(b)-b\alpha D(a))=0$.

(iii) *(a a b-b a a) b D(a a b-b a a)=0.*

(iv) *D(a a a b b)=a b a a D(b)+(a b b+b b a) a D(a)+a a D(a b b-b b a).*

Proof.(i) Replacing *a a b* for *c* in Lemma 2.1 (iii) we have

(a a b-b a a) b D(a a b)=a a (a a b-b a a) b D(b)+b a (a a b-b a a) b D(a). (1)

Then, replacing *a+b* for *b* in (1) we get (i).

(ii) Replacing *a+b* for *a* in (i) and using (1), we obtain

(a a b-b a a) b (D(a a b)-a a D(b)-b a D(a))=0. (2)

(iii) Using Lemma 2.1 (i) and Lemma 2.2 (ii), we have

(a a b-b a a) b (D(b a a)-a a D(b)-b a D(a))=0. (3)

Taking (2) minus (3), we have (iii).

(iv) Replacing *b b a* for *b* in Lemma 2.1 (i), we obtain

D(a a b b a+b b a a a)=2a a D(b b a)+2b b a a D(a) (4)

and replacing *a b b* for *b* in Lemma 2.1 (i), we obtain

D(a a a b +a b b a a)=2a a D(a b b)+2a b b a D(a) (5)

Taking (5) minus (4), we have

D(a a a b -b b a a a)=2a a D(a b b-b b a)+2(a b b-b b a)a a D(a). (6)

Replacing *a b a* for *a* in Lemma 2.1 (i), then we have

D(a a a b b-b b a a a)=2a b a a D(b)+4b a a b D(a). (7)

Taking (6) plus (7) and using *M* is 2-torsion free we have (iv).

Theorem 2.1 Let *M* is a completely prime and 2-torsion free Γ-ring. If there exists a nonzero Jordan left derivation $D:M\rightarrow M$ and $a a bb c = a b ba c$ for all $a,b,c\in M$ and $a , b \in \Gamma$ then *M* is commutative.

Proof. From Lemma 2.2 (iii), *(a a b-b a a) b D(a a b-b a a)=0* for all $b \in \Gamma$. Then, for all $a,b\in M$ and $a \in \Gamma$, *a a b-b a a=0* or *D(a a b-b a a)=0* since *M* is a completely prime Γ-ring. If *a a b-b a a=0*, then *M* is commutative. If *D(a a b-b a a)=0*, then *2D(a a b)=D(a a b)+D(b a a).* Replacing *a b b* for *b*, we obtain *2D(a a a b b)=2a b a a D(b)+4a a b b D(a).* Using Lemma 2.2 (iv) and *M* is 2-torsion free, we have *(a a b-b a a) b D(a)=0* for all $a,b\in M$ and $a , b \in \Gamma$ Then, since $D\neq 0$, *M* is commutative.

Theorem 2.2 If M is a completely prime Γ-ring with 2-torsion free and if $a a bb c = a b ba c$ for all $a,b,c \in M$ and $a, b \in \Gamma$, then a Jordan left derivation on M is a left derivation on M.

Proof. Since M is commutative and 2-torsion free, using Lemma 2.1 (i), the proof is immediately shown.

References

[1] W. E. Barnes: On the Γ-rings of Nobusawa, Pacific J. Math., 18(1966), 411-422.

[2] M. Bresar and J. Vukman: Jordan derivations on prime rings, Bull. Austral. Math. Soc., 37(1988), 321-322.

[3] M. Bresar and J. Vukman: On the left derivations and related mappings, Proc. of the AMS., Vol:110, 1(1990), 7-16.

[4] K.W. Jun and B.D. Kim: A note on Jordan left derivations, Bull. Korean Math. Soc., 33(1996), No:2, 221-228.

[5] N. Nobusawa: On a generalization of the ring theory, Osaka J. Math., 1(1964).

[6] M. Sapanci and A. Nakajima, Jordan derivations on completely prime gamma rings, Math. Japonica, Vol:46. No:1, 1997, 47-51.