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Bessel Potansiyelli Sturm-Liouville Diferensiyel Denklemlerin Çözümleri İçin İntegral Gösterilimleri

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Özet: Bu çalışmada, [1]'de incelenen ve self-adjoint genişlemeleri yazılan Bessel potansiyelli Sturm-Liouville operatörleri için çevirme operatörü tipinde gösterilimler elde edilmiştir.
Anahtar kelimeler: Çevirme operatörü, İntegral denklemi, Sturm-Liouville operatörü

Integral Representations for Solutions of Sturm-Lıouville Differential Equations With Bessel Potential

Abstract: In this study, representations with transformation operator have been obtained for Sturm-Liouville operators with bessel potential which have been written self-adjoint extensions and have been considered in [1].
Key Words: Transformation operator, Integral equation, Sturm-Liouville

1. Giriş

$$-y''+l(l+1)x^{-2}y+q(x)y = l\,y, \quad l = k^2,\ x\in(0,p],\ \left|2l\right|<1 \tag{1}$$

$$\lim_{x\to 0^+} x^{2l}y(x)=0,\ \mathrm{y}(p)=0 \tag{2}$$

$$\begin{pmatrix} y \\ y' \end{pmatrix}(d+0)=A\begin{pmatrix} y \\ y' \end{pmatrix}(d-0),\ A=\begin{pmatrix} a & 0 \\ 0 & a^{-1} \end{pmatrix} \tag{3}$$

problemini ele alalım. Burada $q(x)$ reel değerli fonksiyon, $a>0,\ a\neq 1,\ d\in(0,p]$ şeklindedir.

Aralığın iç noktasında süreksizliğe sahip sınır-değer problemleri matematik, mekanik, fizik ve jeofizik gibi bilim dallarında sıklıkla karşımıza çıkar ve böyle problemler materyalin süreksizlik özelliklerine bağlıdır. Süreksizliğe sahip olmayan diferansiyel operatörlerin ters ve düz spektral problemleri [6]-[10] çalışmalarında incelenmiştir. Süreksizliğin varlığı operatörlerin incelenmesinde temel niteliksel gelişmeler sağlamıştır. Süreksizliğe sahip sınır-değer problemleri için düz ve ters problemlerin çeşitli formülasyonları [11]-[12] ve diğer çalışmalarda ele alınmıştır.

Aralığın iç noktasında singüleriteye ve süreksizlik koşullarına sahip diferansiyel operatörler, R. Kh. Amirov, V. A. Yurko[2] tarafından çalışılmıştır. Bu çalışmada x=0 noktasında singüleriteye sahip self-adjoint olmayan Bessel potansiyelli Sturm-Liouville operatörü için sonlu aralığın iç noktasında çözümün süreksizliğe sahip olduğu durumu incelenmiştir ve verilen operatörün spektral özellikleri ve bu spektral özelliklere göre ters problemin konumu ve çözümü için teklik teoremleri ispatlanmıştır.

Benzer şekilde R. Kh. Amirov [3] çalışmasında, self-adjoint olmayan Bessel potansiyelli Sturm-Liouville operatörü için sonlu aralıkta sonlu sayıda süreksizlik noktalarına sahip olduğu durum incelenmiştir. Burada verilen diferansiyel operatörü üreten diferansiyel denklemin çözümlerinin davranışları, operatörün spektral özellikleri, spektrumu basit olduğu durumda yani yalnızca özdeğerlerden oluştuğu durumda, özdeğerlere karşılık gelen özfonksiyon ve koşulmuş fonksiyonlara göre operatörün ayrılışımı, spektral parametrelere göre ters problemin konumu ve bu ters problemlerin çözümü için teklik teoremleri ispatlanmıştır.

R. Kh. Amirov'un [4] çalışmasında, sonlu aralığın iç noktasında süreksizliğe sahip Sturm-Liouville diferansiyel operatörler sınıfı için ve [5] çalışmasında Dirac operatörü için çevirme operatörü, çekirdek fonksiyonunun bazı özellikleri, spektral karakteristiklerin özellikleri ve ters problem için teklik teoremleri öğrenilmiştir.

2. İntegral Denklemin Oluşturulması

(1) denkleminin $x\to 0^{+}$ iken $y_1(x)=x^{l+1}[1+o(1)],\ y_2(x)=(l+1)x^{l}[1+o(1)]$ koşullarını sağlayan asimptotik çözümleri mevcuttur. Fakat $|2l|<1$ iken $y(0)$ ve $y'(0)$ değerleri mevcut değildir. Dolayısıyla (1) denklemi ve (2) sınır koşulu ve (3) süreksizlik

koşulunun ürettiği operatörün bu ifadelere benzer değerleri de tanımlı olacak şekilde yeni bir operatör tanımlamamız gerekir. Yeni tanımlanan bu operatör, verilen diferansiyel operatörün self-adjoint operatörü olarak alınabilir.

Amirov ve Guseinov [1] çalışmalarında $\ell(y) := -y'' + l(l+1)x^{-2} + \frac{c}{x^a} + q(x))$ diferansiyel ifadesi ve ayrık sınır koşullarının ürettiği operatörler için sınır koşulları dilinde self-adjoint genişlemeleri vermişlerdir. Bu çalışmalarında şu lemmayı ispatlamışlardır. Burada $c \in \mathbb{R},\ |2l| < 1,\ 1 < a < 2,\ q(x) \in L_2^{\mathbb{R}}(0,p)$.

Lemma 1: $y(x) \in D(L^*{}_0)$ olmak üzere,

$$(\Gamma_1 y)(x) = x^{2l} y(x),\ (\Gamma_2 y)(x) = x^{-l-1}[xy'(x) + \ell y(x)]$$

fonksiyonlarının $x \to 0^+$ iken limitleri vardır. Yani,

$$\lim_{x \to 0^+} (\Gamma_i y)(x) = (\Gamma_i y)(0),\ \text{i=1,2.}$$

Burada $L^*{}_0$ verilen L_0 operatörünün eşlenik operatörüdür. L_0 ise $D_0^{'} = C_0^{\infty}(0,p)$ kümesinde tanımlı $L_0^{'} := L_0^{'} y = \ell y$ operatörünün kapanışıdır. Dolayısıyla $L_0^{'}$ operatörü L_0 operatörünün minimal operatörüdür. Belli ki $L_0^{'}$ operatörü $L_2(0,p)$ uzayında simetrik operatördür.

Şimdi $-y'' + l(l+1)x^{-2} y + q(x) y = l\, y$ diferansiyel denklemi için sınır değer problemini yazalım. $|2l| < 1$ iken $y(0)$ ve $y'(0)$ değerleri mevcut olmadığından sınır koşullarını ancak $(\Gamma_1 y)(x)$ ve $(\Gamma_2 y)(x)$ fonksiyonları dilinde verebiliriz**.** Bunun için denklemi $(\Gamma_1 y)(x)$ ve $(\Gamma_2 y)(x)$ fonksiyonları yardımıyla sisteme indirgeyelim.

$-y'' + l(l+1)x^{-2} y + q(x) y = -x^{l}[x^{-l} y' + l x^{-l-1} y]' + q(x) y = l\, y$ eşitliğinde

$$(\Gamma_1 y)(x) = y_1(x) = x^{2l} y(x),\ (\Gamma_2 y)(x) = y_2(x) = x^{-l-1}[xy'(x) + \ell y(x)]$$
$$u_1(x) = x^{-2l} - 1,\ u_2(x) = x^{2l} - 1,\ y_2(x) = k y_3(x)$$

alırsak,

$$\begin{cases} y_3^{'} + k y_1 = -k u_1(x) y_1 + \frac{1}{k} q(x) x^{-2l} y_1 \\ y_1^{'} - k y_3 = k u_2(x) y_3 \end{cases}$$

sistemini elde ederiz.

$$\begin{cases} y_3' + ky_1 = -ku_1(x)y_1 + \frac{1}{k}q(x)x^{-2l}y_1 \\ y_1' - ky_3 = ku_2(x)y_3 \end{cases} \tag{4}$$

$$y_1(x) = \mathrm{y}_1(p)=0 \tag{5}$$

$$\begin{pmatrix} y \\ y' \end{pmatrix}(d+0) = A\begin{pmatrix} y \\ y' \end{pmatrix}(d-0),\ A = \begin{pmatrix} a & 0 \\ 0 & a^{-1} \end{pmatrix} \tag{6}$$

problemini ele alalım. (4) denkleminin $\begin{pmatrix} y_1 \\ y_3 \end{pmatrix}(0) = \begin{pmatrix} 1 \\ i \end{pmatrix}$ başlangıç koşullarını ve (6) süreksizlik koşulunu sağlayan çözümü, $a^+ = \frac{1}{2}\left(a + \frac{1}{a}\right),\ a^- = \frac{1}{2}\left(a - \frac{1}{a}\right)$ olmak üzere,

$x < d$ iken,

$$\begin{pmatrix} y_1 \\ y_3 \end{pmatrix} = \begin{pmatrix} e^{ikx} - k\int_0^x u_1(x)y_1(x)\sin k(x-t)dt + k\int_0^x u_2(x)y_3(x)\cos k(x-t)dt \\ + \frac{1}{k}\int_0^x t^{-2l}q(t)y_1(t)\sin k(x-t)dt \\ ie^{ikx} - k\int_0^x u_1(x)y_1(x)\cos k(x-t)dt + k\int_0^x u_2(x)y_3(x)\sin k(x-t)dt \\ + \frac{1}{k}\int_0^x t^{-2l}q(t)y_1(t)\cos(x-t)dt \end{pmatrix} \tag{7}$$

$x > d$ iken,

$$\begin{aligned} y_1 = a^+e^{ikx} + a^-e^{ik(2d-x)} &- k\int_0^d a^+\sin k(x-t) - a^-\sin k(x+t-2d)u_1(t)y_1(t)dt \\ &+ k\int_0^d a^+\cos k(x-t) + a^-\cos k(x+t-2d)u_2(t)y_3(t)dt \\ &+ \frac{1}{k}\int_0^x (a^+\sin k(x-t) - a^-\sin k(x+t-2d))t^{-2l}q(t)y_1(t)dt \\ &- k\int_d^x (\sin k(x-t)u_1(t)y_1(t) - \cos k(x-t)u_2(t)y_3(t)dt \\ &+ \frac{1}{k}\int_d^x \sin(x-t)t^{-2l}q(t)y_1(t)dt \end{aligned} \tag{8}$$

$$
\begin{aligned}
y_3 = ia^{+}e^{ikx} - ia^{-}e^{ik(2d-x)} &- k\int_0^d a^{+}\cos k(x-t) - a^{-}\cos k(x+t-2d) u_1(t)y_1(t)dt \\
&- k\int_0^d a^{+}\sin k(x-t) + a^{-}\sin k(x+t-2d) u_2(t)y_3(t)dt \\
&+ \frac{1}{k}\int_0^x (a^{+}\cos k(x-t) - a^{-}\cos k(x+t-2d))t^{-2l}q(t)y_1(t)dt \\
&- k\int_d^x (\cos k(x-t)u_1(t)y_1(t) + \sin k(x-t)u_2(t)y_3(t)dt \\
&+ \frac{1}{k}\int_d^x \cos(x-t)t^{-2l}q(t)y_1(t)dt
\end{aligned} \tag{9}
$$

şeklindedir.

Şimdi (4) denkleminin $\begin{pmatrix} y_1 \\ y_3 \end{pmatrix}(0) = \begin{pmatrix} 1 \\ i \end{pmatrix}$ başlangıç koşullarını ve (6) süreksizlik koşulunu sağlayan her bir çözümünün,

$x < d$ iken**,**

$$
\begin{pmatrix} y_1 \\ y_3 \end{pmatrix} = \begin{pmatrix} e^{ikx} + a(x)e^{ikx} + \int_{-x}^{x} K_{11}(x,t)e^{ikt}dt + i\int_{-x}^{x} K_{12}(x,t)e^{ikt}dt \\ ie^{ikx} + ia(x)e^{ikx} + \int_{-x}^{x} K_{21}(x,t)e^{ikt}dt + i\int_{-x}^{x} K_{22}(x,t)e^{ikt}dt \end{pmatrix} \tag{10}
$$

$x > d$ iken**,**

$$
\begin{pmatrix} y_1 \\ y_3 \end{pmatrix} = \begin{pmatrix} y_{10} + a(x)e^{ikx} + b(x)e^{ik(2d-x)} + \int_{-x}^{x} K_{11}(x,t)e^{ikt}dt + i\int_{-x}^{x} K_{12}(x,t)e^{ikt}dt \\ y_{30} + ia(x)e^{ikx} - ib(x)e^{ik(2d-x)} + \int_{-x}^{x} K_{21}(x,t)e^{ikt}dt + i\int_{-x}^{x} K_{22}(x,t)e^{ikt}dt \end{pmatrix} \tag{11}
$$

şeklinde bir integral gösterilime sahip olduğunu ispatlayalım. Burada $\begin{pmatrix} y_{10} \\ y_{30} \end{pmatrix} = \begin{pmatrix} a^{+}e^{ikx} + a^{-}e^{ik(2d-x)} \\ ia^{+}e^{ikx} - ia^{-}e^{ik(2d-x)} \end{pmatrix}$, $K_{ij}(x,t),\ i,j = 1,2.$ fonksiyonları reel değerli, $a(x) = a_1(x) + ia_2(x),\ b(x) = b_1(x) + ib_2(x)$ olmak üzere $a_i(x),\ b_i(x),\ i = 1,2.$ fonksiyonları mutlak sürekli fonksiyonlardır. (10) ve (11) ifadeleri (8) ve (9) çözümünde yerine yazılırsa,

$$a(x)e^{ikx}+b(x)e^{ik(2d-x)}+\int_{-x}^{x}K_{11}(x,t)e^{ikt}dt+i\int_{-x}^{x}K_{12}(x,t)e^{ikt}dt=$$

$$-k\int_{0}^{d}a^{+}\sin k(x-t)-a^{-}\sin k(x+t-2d)u_{1}(t)\Big\{e^{ikt}+a(t)e^{ikt}$$

$$+\int_{-t}^{t}K_{11}(t,s)e^{iks}ds+i\int_{-t}^{t}K_{12}(t,s)e^{iks}ds\Big\}dt$$

$$+k\int_{0}^{d}a^{+}\cos k(x-t)+a^{-}\cos k(x+t-2d)u_{2}(t)\Big\{ie^{ikt}+ia(t)e^{ikt}$$

$$+\int_{-t}^{t}K_{21}(t,s)e^{iks}ds+i\int_{-t}^{t}K_{22}(t,s)e^{iks}ds\Big\}dt$$

$$+\frac{1}{k}\int_{0}^{x}(a^{+}\sin k(x-t)-a^{-}\sin k(x+t-2d))t^{-2l}q(t)\Big\{e^{ikt}+a(t)e^{ikt}$$

$$+\int_{-t}^{t}K_{11}(t,s)e^{iks}ds+i\int_{-t}^{t}K_{12}(t,s)e^{iks}ds\Big\}dt$$

$$-k\int_{d}^{x}(\sin k(x-t)u_{1}(t)dt\ \Big\{a^{+}e^{ikt}+a^{-}e^{ik(2d-t)}+a(t)e^{ikt}+b(t)e^{ik(2d-t)}$$

$$+\int_{-t}^{t}K_{11}(t,s)e^{iks}ds+i\int_{-t}^{t}K_{12}(t,s)e^{iks}ds\Big\}dt$$

$$+k\int_{d}^{x}\cos k(x-t)u_{2}(t)\Big\{ia^{+}e^{ikt}-ia^{-}e^{ik(2d-t)}+ia(t)e^{ikt}-ib(t)e^{ik(2d-t)}$$

$$+\int_{-t}^{t}K_{21}(t,s)e^{iks}ds+i\int_{-t}^{t}K_{22}(t,s)e^{iks}ds\Big\}dt$$

$$+\frac{1}{k}\int_{d}^{x}\sin(x-t)t^{-2l}q(t)\Big\{a^{+}e^{ikt}+a^{-}e^{ik(2d-t)}+a(t)e^{ikt}+b(t)e^{ik(2d-t)}$$

$$+\int_{-t}^{t}K_{11}(t,s)e^{iks}ds+i\int_{-t}^{t}K_{12}(t,s)e^{iks}ds\Big\}dt$$

ve

$$ia(x)e^{ikx}-ib(x)e^{ik(2d-x)}+\int_{-x}^{x}K_{21}(x,t)e^{ikt}dt+i\int_{-x}^{x}K_{22}(x,t)e^{ikt}dt=$$

$$-k\int_{0}^{d}a^{+}\cos k(x-t)-a^{-}\cos k(x+t-2d)u_{1}(t)\left\{e^{ikt}+a(t)e^{ikt}+\int_{-t}^{t}K_{11}(t,s)e^{iks}ds+i\int_{-t}^{t}K_{12}(t,s)e^{iks}ds\right\}dt$$

$$-k\int_{0}^{d}a^{+}\sin k(x-t)+a^{-}\sin k(x+t-2d)u_{2}(t)\left\{ie^{ikt}+ia(t)e^{ikt}+\int_{-t}^{t}K_{21}(t,s)e^{iks}ds+i\int_{-t}^{t}K_{22}(t,s)e^{iks}ds\right\}dt$$

$$
\begin{aligned}
&+\frac{1}{k}\int_0^x (a^+\cos k(x-t)-a^-\cos k(x+t-2d))t^{-2l}q(t)\left\{e^{ikt}+a(t)e^{ikt}+\int_{-t}^{t}K_{11}(t,s)e^{iks}ds+i\int_{-t}^{t}K_{12}(t,s)e^{iks}ds\right\}dt\\
&-k\int_d^x(\cos k(x-t)u_1(t)dt\ \left\{a^+e^{ikt}+a^-e^{ik(2d-t)}+a(t)e^{ikt}+b(t)e^{ik(2d-t)}+\int_{-t}^{t}K_{11}(t,s)e^{iks}ds+i\int_{-t}^{t}K_{12}(t,s)e^{iks}ds\right\}dt\\
&-k\int_d^x\sin k(x-t)u_2(t)\left\{ia^+e^{ikt}-ia^-e^{ik(2d-t)}+ia(t)e^{ikt}-ib(t)e^{ik(2d-t)}+\int_{-t}^{t}K_{21}(t,s)e^{iks}ds+i\int_{-t}^{t}K_{22}(t,s)e^{iks}ds\right\}dt\\
&+\frac{1}{k}\int_d^x\cos(x-t)t^{-2l}q(t)\left\{a^+e^{ikt}+a^-e^{ik(2d-t)}+a(t)e^{ikt}+b(t)e^{ik(2d-t)}+\int_{-t}^{t}K_{11}(t,s)e^{iks}ds+i\int_{-t}^{t}K_{12}(t,s)e^{iks}ds\right\}dt
\end{aligned}
$$

integral denklemleri elde edilir. Gerekli hesaplamalar yapılırsa,

$$
\begin{aligned}
&\int_{-x}^{x}K_{11}(x,t)e^{ikt}dt=\frac{a^+k}{4}\int_{-x}^{2d-x}u_1\left(\tfrac{x+z}{2}\right)a_2\left(\tfrac{x+z}{2}\right)e^{ikz}\,dz+\frac{a^-k}{4}\int_{x-2d}^{x}u_1\left(d-\tfrac{x-z}{2}\right)a_2\left(d-\tfrac{x-z}{2}\right)e^{ikz}\,dz\\
&-\frac{a^+}{4k}\int_{-x}^{2d-x}\left(\tfrac{x+z}{2}\right)^{-2l}q\left(\tfrac{x+z}{2}\right)a_2\left(\tfrac{x+z}{2}\right)e^{ikz}\,dz-\frac{a^-}{4k}\int_{x-2d}^{x}\left(d-\tfrac{x-z}{2}\right)^{-2l}q\left(d-\tfrac{x-z}{2}\right)^{-2l}a_2\left(d-\tfrac{x-z}{2}\right)^{-2l}e^{ikz}\,dz\\
&-\frac{a^+k}{2}\int_{x-2d}^{x}\left\{\int_{\frac{x-z}{2}}^{d}K_{12}(t,z+t-x)u_1(t)dt\right\}e^{ikz}\,dz+\frac{a^+k}{2}\int_{-x}^{2d-x}\left\{\int_{\frac{x+z}{2}}^{d}K_{12}(t,z+x-t)u_1(t)dt\right\}e^{ikz}\,dz\\
&+\frac{a^-k}{2}\int_{x-2d}^{x}\left\{\int_{d-\frac{x-z}{2}}^{d}K_{12}(t,z+2d-x-t)u_1(t)dt\right\}e^{ikz}\,dz-\frac{a^-k}{2}\int_{-x}^{2d-x}\left\{\int_{d-\frac{x+z}{2}}^{d}K_{12}(t,z+x+t-2d)u_1(t)dt\right\}e^{ikz}\,dz\\
&-\frac{a^+k}{4}\int_{-x}^{2d-x}u_2\left(\tfrac{x+z}{2}\right)a_2\left(\tfrac{x+z}{2}\right)e^{ikz}\,dz-\frac{a^-k}{4}\int_{x-2d}^{x}u_2\left(d-\tfrac{x-z}{2}\right)a_2\left(d-\tfrac{x-z}{2}\right)e^{ikz}\,dz\\
&+\frac{a^+k}{2}\int_{x-2d}^{x}\left\{\int_{\frac{x-z}{2}}^{d}K_{21}(t,z+t-x)u_2(t)dt\right\}e^{ikz}\,dz+\frac{a^+k}{2}\int_{-x}^{2d-x}\left\{\int_{\frac{x+z}{2}}^{d}K_{21}(t,z+x-t)u_2(t)dt\right\}e^{ikz}\,dz\\
&+\frac{a^-k}{2}\int_{x-2d}^{x}\left\{\int_{d-\frac{x-z}{2}}^{d}K_{21}(t,z+2d-x-t)u_2(t)dt\right\}e^{ikz}\,dz+\frac{a^-k}{2}\int_{-x}^{2d-x}\left\{\int_{d-\frac{x+z}{2}}^{d}K_{21}(t,z+x+t-2d)u_2(t)dt\right\}e^{ikz}\,dz\\
&+\frac{a^+}{2}\int_{-x}^{x}\left\{\int_0^d q(s)s^{-2l}\int_{t-x+s}^{t+x-s}K_{11}(s,z)dz\,ds\right\}e^{ikt}dt-\frac{a^-}{2}\int_{-x}^{x}\left\{\int_0^d q(s)s^{-2l}\int_{t-x-s+2d}^{t+x+s-2d}K_{11}(s,z)dz\,ds\right\}e^{ikt}dt\\
&+\frac{k}{4}\int_{2d-x}^{x}u_1\left(\tfrac{x+z}{2}\right)a_2\left(\tfrac{x+z}{2}\right)e^{ikz}\,dz+\frac{k}{4}\int_{x}^{2d-x}u_1\left(d-\tfrac{x-z}{2}\right)b_2\left(d-\tfrac{x-z}{2}\right)e^{ikz}\,dz\\
&-\frac{k}{2}\int_{-x}^{x-2d}\left\{\int_{\frac{x-z}{2}}^{x}K_{12}(t,z+t-x)u_1(t)dt\right\}e^{ikz}\,dz-\frac{k}{2}\int_{x-2d}^{x}\left\{\int_d^x K_{12}(t,z+t-x)u_1(t)dt\right\}e^{ikz}\,dz\\
&-\frac{k}{2}\int_{-x}^{2d-x}\left\{\int_d^x K_{12}(t,z+x-t)u_1(t)dt\right\}e^{ikz}\,dz+\frac{k}{2}\int_{2d-x}^{x}\left\{\int_{\frac{x+z}{2}}^{x}K_{12}(t,z+x-t)u_1(t)dt\right\}e^{ikz}\,dz-
\end{aligned}
$$

$$-\frac{k}{4}\int_{2d-x}^{x} u_2\left(\tfrac{x+z}{2}\right)a_2\left(\tfrac{x+z}{2}\right)e^{ikz}\,dz-\frac{k}{4}\int_{x}^{2d-x} u_2\left(d+\tfrac{x-z}{2}\right)b_2\left(d+\tfrac{x-z}{2}\right)e^{ikz}\,dz$$

$$+\frac{k}{2}\int_{-x}^{x-2d}\left\{\int_{\frac{x-z}{2}}^{x} K_{21}(t,z+t-x)u_2(t)dt\right\}e^{ikz}\,dz+\frac{k}{2}\int_{x-2d}^{x}\left\{\int_{d}^{x} K_{21}(t,z+t-x)u_2(t)dt\right\}e^{ikz}\,dz$$

$$+\frac{k}{2}\int_{-x}^{2d-x}\left\{\int_{d}^{x} K_{21}(t,z+x-t)u_2(t)dt\right\}e^{ikz}\,dz+\frac{k}{2}\int_{2d-x}^{x}\left\{\int_{\frac{x+z}{2}}^{x} K_{21}(t,z+x-t)u_2(t)dt\right\}e^{ikz}\,dz-$$

$$-\frac{1}{4k}\int_{2d-x}^{x}\left(\tfrac{x+z}{2}\right)^{-2l} q\left(\tfrac{x+z}{2}\right)a_2\left(\tfrac{x+z}{2}\right)e^{ikz}\,dz+\frac{1}{2}\int_{-x}^{x}\left\{\int_{d}^{x} q(s)s^{-2l}\int_{t-x+s}^{t+x-s} K_{11}(s,z)dz\,ds\right\}e^{ikt}dt$$

$$\int_{-x}^{x} K_{12}(x,t)e^{ikt}dt=-\frac{a^+k}{4}\int_{-x}^{2d-x} u_1\left(\tfrac{x+z}{2}\right)e^{ikz}\,dz-\frac{a^-k}{4}\int_{x-2d}^{x} u_1\left(d-\tfrac{x-z}{2}\right)e^{ikz}\,dz$$

$$-\frac{a^+k}{4}\int_{-x}^{2d-x} u_1\left(\tfrac{x+z}{2}\right)a_1\left(\tfrac{x+z}{2}\right)e^{ikz}\,dz-\frac{a^-k}{4}\int_{x-2d}^{x} u_1\left(d-\tfrac{x-z}{2}\right)^{-2l} a_1\left(d-\tfrac{x-z}{2}\right)e^{ikz}\,dz$$

$$+\frac{a^+}{4k}\int_{-x}^{2d-x}\left(\tfrac{x+z}{2}\right)^{-2l} q\left(\tfrac{x+z}{2}\right)a_1\left(\tfrac{x+z}{2}\right)e^{ikz}\,dz+\frac{a^-}{4k}\int_{x-2d}^{x}\left(d-\tfrac{x-z}{2}\right)^{-2l} q\left(d-\tfrac{x-z}{2}\right)a_1\left(d-\tfrac{x-z}{2}\right)e^{ikz}\,dz$$

$$+\frac{a^+k}{2}\int_{x-2d}^{x}\left\{\int_{\frac{x-z}{2}}^{d} K_{11}(t,z+t-x)u_1(t)dt\right\}e^{ikz}\,dz-\frac{a^+k}{2}\int_{-x}^{2d-x}\left\{\int_{\frac{x+z}{2}}^{d} K_{11}(t,z+x-t)u_1(t)dt\right\}e^{ikz}\,dz$$

$$-\frac{a^-k}{2}\int_{x-2d}^{x}\left\{\int_{d-\frac{x-z}{2}}^{d} K_{11}(t,z+2d-x-t)u_1(t)dt\right\}e^{ikz}\,dz$$

$$+\frac{a^-k}{2}\int_{-x}^{2d-x}\left\{\int_{d-\frac{x+z}{2}}^{d} K_{11}(t,z+x+t-2d)u_1(t)dt\right\}e^{ikz}\,dz$$

$$+\frac{a^+k}{4}\int_{-x}^{2d-x} u_2\left(\tfrac{x+z}{2}\right)e^{ikz}\,dz+\frac{a^-k}{4}\int_{x-2d}^{x} u_2\left(d-\tfrac{x-z}{2}\right)e^{ikz}\,dz+\frac{a^+k}{4}\int_{-x}^{2d-x} u_2\left(\tfrac{x+z}{2}\right)a_1\left(\tfrac{x+z}{2}\right)e^{ikz}\,dz$$

$$+\frac{a^+k}{2}\int_{-x}^{2d-x}\left\{\int_{\frac{x+z}{2}}^{d} K_{22}(t,z+x-t)u_2(t)dt\right\}e^{ikz}\,dz+$$

$$+\frac{a^-k}{2}\int_{-x}^{2d-x}\left\{\int_{d-\frac{x+z}{2}}^{d}K_{22}(t,z+x+t-2d)u_2(t)dt\right\}e^{ikz}dz+\frac{a^+}{4k}\int_{-x}^{2d-x}\left(\tfrac{x+z}{2}\right)^{-2l}q\left(\tfrac{x+z}{2}\right)e^{ikz}dz$$

$$+\frac{a^-}{4k}\int_{x-2d}^{x}\left(d-\tfrac{x-z}{2}\right)^{-2l}q\left(d-\tfrac{x-z}{2}\right)e^{ikz}dz+\frac{a^+}{2}\int_{-x}^{x}\left\{\int_0^d q(s)s^{-2l}\int_{t-x+s}^{t+x-s}K_{12}(s,z)dz\,ds\right\}e^{ikt}dt$$

$$-\frac{a^-}{2}\int_{-x}^{x}\left\{\int_0^d q(s)s^{-2l}\int_{t-x-s+2d}^{t+x+s-2d}K_{12}(s,z)dz\,ds\right\}e^{ikt}dt$$

$$-\frac{a^+k}{4}\int_{2d-x}^{x}u_1\left(\tfrac{x+z}{2}\right)e^{ikz}dz-\frac{a^-k}{4}\int_{x}^{2d-x}u_1\left(d-\tfrac{x-z}{2}\right)e^{ikz}dz-\frac{k}{4}\int_{2d-x}^{x}u_1\left(\tfrac{x+z}{2}\right)a_1\left(\tfrac{x+z}{2}\right)e^{ikz}dz$$

$$-\frac{k}{4}\int_{x}^{2d-x}u_1\left(d-\tfrac{x-z}{2}\right)b_2\left(d-\tfrac{x-z}{2}\right)e^{ikz}dz+\frac{k}{2}\int_{-x}^{x-2d}\left\{\int_{\frac{x-z}{2}}^{x}K_{11}(t,z+x-t)u_1(t)dt\right\}e^{ikz}dz+$$

$$+\frac{k}{2}\int_{x-2d}^{x}\left\{\int_d^x K_{11}(t,z+x-t)u_1(t)dt\right\}e^{ikz}dz-\frac{k}{2}\int_{-x}^{2d-x}\left\{\int_d^x K_{11}(t,z+x-t)u_1(t)dt\right\}e^{ikz}dz+$$

$$-\frac{k}{2}\int_{2d-x}^{x}\left\{\int_{\frac{x+z}{2}}^{x}K_{11}(t,z+x-t)u_1(t)dt\right\}e^{ikz}dz+\frac{k}{2}\int_{-x}^{2d-x}\left\{\int_d^x K_{12}(t,z+x-t)u_1(t)dt\right\}e^{ikz}dz+$$

$$+\frac{k}{2}\int_{2d-x}^{x}\left\{\int_{\frac{x+z}{2}}^{x}K_{12}(t,z+x-t)u_1(t)dt\right\}e^{ikz}dz+\frac{a^+k}{4}\int_{2d-x}^{x}u_2\left(\tfrac{x+z}{2}\right)e^{ikz}dz+\frac{a^-k}{4}\int_{x}^{2d-x}u_2\left(d+\tfrac{x-z}{2}\right)e^{ikz}dz$$

$$+\frac{k}{4}\int_{2d-x}^{x}u_2\left(\tfrac{x+z}{2}\right)a_1\left(\tfrac{x+z}{2}\right)e^{ikz}dz+\frac{k}{4}\int_{x}^{2d-x}u_2\left(d+\tfrac{x-z}{2}\right)b_1\left(d+\tfrac{x-z}{2}\right)e^{ikz}dz+\frac{k}{2}\int_{-x}^{2d-x}\left\{\int_{\frac{x-z}{2}}^{x}K_{22}(t,z+t-x)u_2(t)dt\right\}e^{ikz}dz+$$

$$+\frac{a^-k}{2}\int_{x-2d}^{x}\left\{\int_{d-\frac{x+z}{2}}^{d}K_{22}(t,z+2d-x-t)u_2(t)dt\right\}e^{ikz}dz$$

$$+\frac{k}{2}\int_{x-2d}^{x}\left\{\int_d^x K_{22}(t,z+t-x)u_2(t)dt\right\}e^{ikz}dz+\frac{k}{2}\int_{-x}^{2d-x}\left\{\int_d^x K_{22}(t,z+x-t)u_2(t)dt\right\}e^{ikz}dz$$

$$+\frac{k}{2}\int_{2d-x}^{x}\left\{\int_{\frac{x+z}{2}}^{x}K_{22}(t,z+x-t)u_2(t)dt\right\}e^{ikz}dz+\frac{a^+}{4k}\int_{2d-x}^{x}\left(\tfrac{x+z}{2}\right)^{-2l}q\left(\tfrac{x+z}{2}\right)e^{ikz}dz+\frac{a^-}{4k}\int_{x}^{2d-x}\left(d+\tfrac{x-z}{2}\right)^{-2l}q\left(d+\tfrac{x-z}{2}\right)e^{ikz}dz$$

$$+\frac{1}{4k}\int_{2d-x}^{x}\left(\tfrac{x+z}{2}\right)^{-2l}q\left(\tfrac{x+z}{2}\right)a_1\left(\tfrac{x+z}{2}\right)e^{ikz}dz+\frac{1}{4k}\int_{x}^{2d-x}\left(d+\tfrac{x-z}{2}\right)^{-2l}q\left(d+\tfrac{x-z}{2}\right)b_1\left(d+\tfrac{x-z}{2}\right)e^{ikz}dz+$$

$$+\frac{1}{2}\int_{-x}^{x}\left\{\int_d^x q(s)s^{-2l}\int_{t-x+s}^{t+x-s}K_{12}(s,z)dz\,ds\right\}e^{ikt}dt$$

$$\int_{-x}^{x} K_{21}(x,t)e^{ikt}dt = -\frac{a^+k}{4}\int_{-x}^{2d-x} u_1\left(\frac{x+z}{2}\right)e^{ikz}dz - \frac{a^-k}{4}\int_{x-2d}^{x} u_1\left(d-\frac{x-z}{2}\right)e^{ikz}dz - \frac{a^+k}{4}\int_{-x}^{2d-x} u_1\left(\frac{x+z}{2}\right)a_1\left(\frac{x+z}{2}\right)e^{ikz}dz$$

$$+\frac{a^-k}{4}\int_{x-2d}^{x} u_1\left(d-\frac{x-z}{2}\right)a_1\left(d-\frac{x-z}{2}\right)e^{ikz}dz + \frac{a^+}{4k}\int_{-x}^{2d-x}\left(\frac{x+z}{2}\right)^{-2l} q\left(\frac{x+z}{2}\right)a_1\left(\frac{x+z}{2}\right)e^{ikz}dz$$

$$-\frac{a^-}{4k}\int_{x-2d}^{x}\left(d-\frac{x-z}{2}\right)^{-2l} q\left(d-\frac{x-z}{2}\right)^{-2l} a_1\left(d-\frac{x-z}{2}\right)^{-2l} e^{ikz}dz - \frac{a^+k}{2}\int_{x-2d}^{x}\left\{\int_{\frac{x-z}{2}}^{d} K_{11}(t,z+t-x)u_1(t)dt\right\}e^{ikz}dz -$$

$$-\frac{a^+k}{2}\int_{-x}^{2d-x}\left\{\int_{\frac{x+z}{2}}^{d} K_{11}(t,z+x-t)u_1(t)dt\right\}e^{ikz}dz + \frac{a^-k}{2}\int_{x-2d}^{x}\left\{\int_{d-\frac{x-z}{2}}^{d} K_{11}(t,z+2d-x-t)u_1(t)dt\right\}e^{ikz}dz +$$

$$+\frac{a^-k}{2}\int_{-x}^{2d-x}\left\{\int_{d-\frac{x+z}{2}}^{d} K_{11}(t,z+x+t-2d)u_1(t)dt\right\}e^{ikz}dz + \frac{a^+k}{4}\int_{-x}^{2d-x} u_2\left(\frac{x+z}{2}\right)a_2\left(\frac{x+z}{2}\right)e^{ikz}dz - \frac{a^-k}{4}\int_{x-2d}^{x} u_2\left(d-\frac{x-z}{2}\right)e^{ikz}dz$$

$$+\frac{a^+k}{4}\int_{-x}^{2d-x} u_2\left(\frac{x+z}{2}\right)a_1\left(\frac{x+z}{2}\right)e^{ikz}dz - \frac{a^-k}{4}\int_{x-2d}^{x} u_2\left(d-\frac{x-z}{2}\right)a_1\left(d-\frac{x-z}{2}\right)e^{ikz}dz - \frac{a^+k}{2}\int_{x-2d}^{x}\left\{\int_{\frac{x-z}{2}}^{d} K_{22}(t,z+t-x)u_2(t)dt\right\}e^{ikz}dz$$

$$+\frac{a^+k}{2}\int_{-x}^{2d-x}\left\{\int_{\frac{x+z}{2}}^{d} K_{22}(t,z+x-t)u_2(t)dt\right\}e^{ikz}dz - \frac{a^-k}{2}\int_{x-2d}^{x}\left\{\int_{d-\frac{x-z}{2}}^{d} K_{22}(t,z+2d-x-t)u_2(t)dt\right\}e^{ikz}dz +$$

$$\frac{a^-k}{2}\int_{-x}^{2d-x}\left\{\int_{d-\frac{x+z}{2}}^{d} K_{22}(t,z+x+t-2d)u_2(t)dt\right\}e^{ikz}dz + \frac{a^+}{4k}\int_{-x}^{2d-x}\left(\frac{x+z}{2}\right)^{-2l} q\left(\frac{x+z}{2}\right)e^{ikz}dz$$

$$-\frac{a^-}{4k}\int_{x-2d}^{x}\left(d-\frac{x-z}{2}\right)^{-2l} q\left(d-\frac{x-z}{2}\right)^{-2l} e^{ikz}dz + \frac{a^+}{2k}\int_{x-2d}^{x}\left\{\int_{\frac{x-z}{2}}^{x} K_{11}(t,z+t-x)q(t)t^{-2l}dt\right\}e^{ikz}dz$$

$$+\frac{a^+}{2k}\int_{-x}^{2d-x}\left\{\int_{\frac{x+z}{2}}^{d} K_{11}(t,z+x-t)q(t)t^{-2l}dt\right\}e^{ikz}dz - \frac{a^-}{2k}\int_{x-2d}^{x}\left\{\int_{d-\frac{x-z}{2}}^{d} K_{11}(t,z+2d-x-t)q(t)t^{-2l}dt\right\}e^{ikz}dz$$

$$-\frac{a^-}{2k}\int_{-x}^{2d-x}\left\{\int_{d-\frac{x+z}{2}}^{d} K_{11}(t,z+x+t-2)q(t)t^{-2l}dt\right\}e^{ikz}dz - \frac{a^+k}{4}\int_{2d-x}^{x} u_1\left(\frac{x+z}{2}\right)a_2\left(\frac{x+z}{2}\right)e^{ikz}dz$$

$$+\frac{a^-k}{4}\int_{x}^{2d-x} u_1\left(d+\frac{x-z}{2}\right)e^{ikz}dz - \frac{k}{4}\int_{2d-x}^{x} u_1\left(\frac{x+z}{2}\right)a_1\left(\frac{x+z}{2}\right)e^{ikz}dz + \frac{k}{4}\int_{x}^{2d-x} u_1\left(d+\frac{x-z}{2}\right)b_1\left(d+\frac{x-z}{2}\right)e^{ikz}dz$$

$$-\frac{k}{2}\int_{-x}^{2d-x}\left\{\int_{\frac{x-z}{2}}^{x} K_{11}(t,z+t-x)u_1(t)dt\right\}e^{ikz}dz - \frac{k}{2}\int_{x-2d}^{x}\left\{\int_{d}^{x} K_{11}(t,z+t-x)u_1(t)dt\right\}e^{ikz}dz$$

$$-\frac{k}{2}\int_{-x}^{2d-x}\left\{\int_{d}^{x}K_{11}(t,z+x-t)u_1(t)dt\right\}e^{ikz}dz-\frac{k}{2}\int_{2d-x}^{x}\left\{\int_{\frac{x+z}{2}}^{x}K_{11}(t,z+x-t)u_1(t)dt\right\}e^{ikz}dz+$$

$$\frac{a^{+}k}{4}\int_{2d-x}^{x}u_2\left(\tfrac{x+z}{2}\right)e^{ikz}dz-\frac{a^{-}k}{4}\int_{x}^{2d-x}u_2\left(d+\tfrac{x-z}{2}\right)e^{ikz}dz$$

$$+\frac{k}{4}\int_{2d-x}^{x}u_2\left(\tfrac{x+z}{2}\right)a_1\left(\tfrac{x+z}{2}\right)e^{ikz}dz-\frac{k}{4}\int_{x}^{2d-x}u_2\left(d+\tfrac{x-z}{2}\right)b_1\left(d+\tfrac{x-z}{2}\right)e^{ikz}dz$$

$$-\frac{k}{2}\int_{-x}^{x-2d}\left\{\int_{\frac{x-z}{2}}^{x}K_{22}(t,z+t-x)u_2(t)dt\right\}e^{ikz}dz-\frac{k}{2}\int_{x-2d}^{x}\left\{\int_{d}^{x}K_{22}(t,z+t-x)u_2(t)dt\right\}e^{ikz}dz$$

$$+\frac{k}{2}\int_{-x}^{2d-x}\left\{\int_{d}^{x}K_{22}(t,z+x-t)u_2(t)dt\right\}e^{ikz}dz+\frac{k}{2}\int_{2d-x}^{x}\left\{\int_{\frac{x+z}{2}}^{x}K_{22}(t,z+x-t)u_2(t)dt\right\}e^{ikz}dz$$

$$+\frac{a^{+}}{4k}\int_{2d-x}^{x}\left(\tfrac{x+z}{2}\right)^{-2l}q\left(\tfrac{x+z}{2}\right)e^{ikz}dz-\frac{a^{-}}{4k}\int_{x}^{2d-x}\left(d+\tfrac{x-z}{2}\right)^{-2l}q\left(d+\tfrac{x-z}{2}\right)e^{ikz}dz+$$

$$\frac{1}{4k}\int_{2d-x}^{x}\left(\tfrac{x+z}{2}\right)^{-2l}q\left(\tfrac{x+z}{2}\right)a_1\left(\tfrac{x+z}{2}\right)e^{ikz}dz-\frac{1}{4k}\int_{x}^{2d-x}\left(d+\tfrac{x-z}{2}\right)^{-2l}q\left(d+\tfrac{x-z}{2}\right)b_1\left(d+\tfrac{x-z}{2}\right)e^{ikz}dz$$

$$+\frac{1}{2k}\int_{-x}^{x-2d}\left\{\int_{\frac{x-z}{2}}^{x}K_{11}(t,z+t-x)t^{-2l}q(t)dt\right\}e^{ikz}dz+\frac{1}{2k}\int_{x-2d}^{x}\left\{\int_{d}^{x}K_{11}(t,z+t-x)t^{-2l}q(t)dt\right\}e^{ikz}dz$$

$$+\frac{1}{2k}\int_{-x}^{2d-x}\left\{\int_{d}^{x}K_{11}(t,z+x-t)t^{-2l}q(t)dt\right\}e^{ikz}dz+\frac{1}{2k}\int_{2d-x}^{x}\left\{\int_{\frac{x+z}{2}}^{x}K_{11}(t,z+x-t)t^{-2l}q(t)dt\right\}e^{ikz}dz$$

$$-\frac{k}{2}\int_{-x}^{x-2d}\left\{\int_{\frac{x-z}{2}}^{x}K_{12}(t,z+t-x)u_1(t)dt\right\}e^{ikz}dz-\frac{k}{2}\int_{x-2d}^{x}\left\{\int_{d}^{x}K_{12}(t,z+t-x)u_1(t)dt\right\}e^{ikz}dz$$

$$-\frac{k}{2}\int_{-x}^{2d-x}\left\{\int_{d}^{x}K_{12}(t,z+x-t)u_1(t)dt\right\}e^{ikz}dz-\frac{k}{2}\int_{2d-x}^{x}\left\{\int_{\frac{x+z}{2}}^{x}K_{12}(t,z+x-t)u_1(t)dt\right\}e^{ikz}dz$$

$$+\frac{a^{+}k}{4}\int_{-x}^{2d-x}u_2\left(\tfrac{x+z}{2}\right)a_2\left(\tfrac{x+z}{2}\right)e^{ikz}dz+\frac{a^{-}k}{4}\int_{x-2d}^{x}u_2\left(d-\tfrac{x-z}{2}\right)a_2\left(d-\tfrac{x-z}{2}\right)e^{ikz}dz$$

$$+\frac{k}{4}\int_{2d-x}^{x}u_2\left(\tfrac{x+z}{2}\right)a_1\left(\tfrac{x+z}{2}\right)e^{ikz}dz$$

$$\int_{-x}^{x} K_{22}(x,t)e^{ikt}dt = -\frac{a^+k}{4}\int_{-x}^{2d-x} u_1\left(\tfrac{x+z}{2}\right)a_2\left(\tfrac{x+z}{2}\right)e^{ikz}\,dz + \frac{a^-k}{4}\int_{-x}^{2d-x} u_1\left(d-\tfrac{x-z}{2}\right)a_2\left(d-\tfrac{x-z}{2}\right)e^{ikz}\,dz$$

$$-\frac{a^-}{4k}\int_{-x}^{2d-x}\left(d-\tfrac{x-z}{2}\right)^{-2l} q\left(d-\tfrac{x-z}{2}\right)a_2\left(d-\tfrac{x-z}{2}\right)e^{ikz}\,dz - \frac{a^+k}{2}\int_{x-2d}^{x}\left\{\int_{\frac{x-z}{2}}^{d} K_{12}(t,z+t-x)u_1(t)dt\right\}e^{ikz}\,dz$$

$$-\frac{a^+k}{2}\int_{-x}^{2d-x}\left\{\int_{\frac{x+z}{2}}^{d} K_{12}(t,z+x-t)u_1(t)dt\right\}e^{ikz}\,dz + \frac{a^-k}{2}\int_{x-2d}^{x}\left\{\int_{d-\frac{x-z}{2}}^{d} K_{12}(t,z+2d-x-t)u_1(t)dt\right\}e^{ikz}\,dz$$

$$+\frac{a^-k}{2}\int_{-x}^{2d-x}\left\{\int_{d-\frac{x+z}{2}}^{d} K_{12}(t,z+x+t-2d)u_1(t)dt\right\}e^{ikz}\,dz + \frac{a^+k}{2}\int_{x-2d}^{x}\left\{\int_{\frac{x-z}{2}}^{d} K_{21}(t,z+t-x)u_2(t)dt\right\}e^{ikz}\,dz$$

$$-\frac{a^+k}{2}\int_{-x}^{2d-x}\left\{\int_{\frac{x+z}{2}}^{d} K_{21}(t,z+x-t)u_2(t)dt\right\}e^{ikz}\,dz + \frac{a^-k}{2}\int_{x-2d}^{x}\left\{\int_{d-\frac{x-z}{2}}^{d} K_{21}(t,z+2d-x-t)u_2(t)dt\right\}e^{ikz}\,dz$$

$$-\frac{a^-k}{2}\int_{-x}^{2d-x}\left\{\int_{d-\frac{x+z}{2}}^{d} K_{21}(t,z+x+t-2d)u_2(t)dt\right\}e^{ikz}\,dz + \frac{a^+}{2k}\int_{x-2d}^{x}\left\{\int_{\frac{x-z}{2}}^{d} K_{12}(t,z+t-x)q(t)t^{-2l}dt\right\}e^{ikz}\,dz$$

$$+\frac{a^+}{2k}\int_{-x}^{2d-x}\left\{\int_{\frac{x+z}{2}}^{d} K_{12}(t,z+x-t)q(t)t^{-2l}dt\right\}e^{ikz}\,dz - \frac{a^-}{2k}\int_{x-2d}^{x}\left\{\int_{d-\frac{x-z}{2}}^{d} K_{12}(t,z+2d-x-t)q(t)t^{-2l}dt\right\}e^{ikz}\,dz$$

$$-\frac{a^-}{2k}\int_{-x}^{2d-x}\left\{\int_{d-\frac{x+z}{2}}^{d} K_{12}(t,z+x+t2d)q(t)t^{-2l}dt\right\}e^{ikz}\,dz - \frac{k}{4}\int_{2d-x}^{x} u_1\left(\tfrac{x+z}{2}\right)a_2\left(\tfrac{x+z}{2}\right)e^{ikz}\,dz$$

$$+\frac{k}{4}\int_{x}^{2d-x} u_1\left(d+\tfrac{x-z}{2}\right)b_2\left(d+\tfrac{x-z}{2}\right)e^{ikz}\,dz - \frac{k}{2}\int_{-x}^{x-2d}\left\{\int_{\frac{x-z}{2}}^{x} K_{12}(t,z+t-x)u_1(t)dt\right\}e^{ikz}\,dz$$

$$-\frac{k}{2}\int_{x-2d}^{x}\left\{\int_{d}^{x} K_{12}(t,z+t-x)u_1(t)dt\right\}e^{ikz}\,dz - \frac{k}{2}\int_{-x}^{2d-x}\left\{\int_{d}^{x} K_{212}(t,z+x-t)u_1(t)dt\right\}e^{ikz}\,dz$$

$$-\frac{k}{2}\int_{2d-x}^{x}\left\{\int_{\frac{x+z}{2}}^{x} K_{12}(t,z+x-t)u_1(t)dt\right\}e^{ikz}\,dz + \frac{a^+k}{4}\int_{-x}^{2d-x} u_2\left(\tfrac{x+z}{2}\right)a_2\left(\tfrac{x+z}{2}\right)e^{ikz}\,dz$$

$$-\frac{a^-k}{4}\int_{x-2d}^{x} u_2\left(d-\tfrac{x-z}{2}\right)a_2\left(d-\tfrac{x-z}{2}\right)e^{ikz}\,dz + \frac{k}{2}\int_{-x}^{x-2d}\left\{\int_{\frac{x-z}{2}}^{x} K_{21}(t,z+t-x)u_2(t)dt\right\}e^{ikz}\,dz$$

$$+\frac{k}{2}\int_{x-2d}^{x}\left\{\int_{d}^{x} K_{21}(t,z+t-x)u_2(t)dt\right\}e^{ikz}\,dz - \frac{k}{2}\int_{-x}^{2d-x}\left\{\int_{d}^{x} K_{21}(t,z+x-t)u_2(t)dt\right\}e^{ikz}\,dz$$

$$-\frac{k}{2}\int_{2d-x}^{x}\left\{\int_{\frac{x+z}{2}}^{x} K_{21}(t,z+x-t)u_2(t)dt\right\}e^{ikz}\,dz - \frac{k}{2}\int_{-x}^{x-2d}\left\{\int_{\frac{x-z}{2}}^{x} K_{22}(t,z+t-x)u_2(t)dt\right\}e^{ikz}\,dz$$

$$-\frac{k}{2}\int_{x-2d}^{x}\left\{\int_{d}^{x} K_{22}(t,z+t-x)u_2(t)dt\right\}e^{ikz}\,dz$$

$$+\frac{1}{4k}\int_{2d-x}^{x}\left(\tfrac{x+z}{2}\right)^{-2l} q\left(\tfrac{x+z}{2}\right)a_2\left(\tfrac{x+z}{2}\right)e^{ikz}\,dz-\frac{1}{4k}\int_{x}^{2d-x}\left(d+\tfrac{x-z}{2}\right)^{-2l} q\left(d+\tfrac{x-z}{2}\right)^{-2l} b_2\left(d+\tfrac{x-z}{2}\right)e^{ikz}\,dz$$

$$+\frac{1}{2k}\int_{-x}^{x-2d}\left\{\int_{\frac{x-z}{2}}^{x}K_{12}(t,z+t-x)t^{-2l}q(t)dt\right\}e^{ikz}\,dz+\frac{1}{2k}\int_{x-2d}^{x}\left\{\int_{d}^{x}K_{12}(t,z+t-x)t^{-2l}q(t)dt\right\}e^{ikz}\,dz$$

$$+\frac{1}{2k}\int_{-x}^{2d-x}\left\{\int_{d}^{x}K_{12}(t,z+x-t)t^{-2l}q(t)dt\right\}e^{ikz}\,dz+\frac{1}{2k}\int_{2d-x}^{x}\left\{\int_{\frac{x+z}{2}}^{x}K_{12}(t,z+x-t)t^{-2l}q(t)dt\right\}e^{ikz}\,dz$$

$$a_1(x)=-\frac{a^+k}{2}\int_0^d u_2(t)a_2(t)dt-\frac{k}{2}\int_d^x u_1(t)a_2(t)dt-\frac{k}{2}\int_d^x u_2(t)a_2(t)dt+\frac{1}{2k}\int_d^x t^{-2l}q(t)a_2(t)dt$$
$$-\frac{a^+k}{2}\int_0^d u_1(t)a_2(t)dt+\frac{a^+}{2k}\int_0^d t^{-2l}q(t)a_2(t)dt$$

$$a_2(x)=\frac{a^+k}{2}\int_0^d u_1(t)dt+\frac{a^+k}{2}\int_0^d u_2(t)dt+\frac{a^+k}{2}\int_0^d u_2(t)a_1(t)dt-\frac{a^+}{2k}\int_0^d t^{-2l}q(t)dt+\frac{a^+k}{2}\int_d^x u_1(t)dt$$
$$+\frac{k}{2}\int_d^x u_1(t)a_1(t)dt+\frac{a^+k}{2}\int_d^x u_2(t)dt+\frac{k}{2}\int_d^x u_2(t)a_1(t)dt-\frac{a^+}{2k}\int_d^x t^{-2l}q(t)dt-\frac{1}{2k}\int_d^x t^{-2l}q(t)a_1(t)dt$$
$$+\frac{a^+k}{2}\int_d^x u_1(t)a_1(t)dt-\frac{a^+}{2k}\int_0^d t^{-2l}q(t)a_1(t)dt-\frac{k}{2}\int_d^x u_1(t)a_2(t)dt-\frac{k}{2}\int_d^x u_2(t)a_2(t)dt+\frac{1}{2k}\int_d^x t^{-2l}q(t)a_2(t)dt$$
$$-\frac{a^+k}{2}\int_0^d u_1(t)a_2(t)dt-\frac{a^+}{2k}\int_0^d t^{-2l}q(t)a_2(t)dt$$

$$b_1(x)=-\frac{a^-k}{2}\int_0^d u_2(t)a_2(t)dt+\frac{k}{2}\int_d^x u_1(t)b_2(t)dt+\frac{k}{2}\int_d^x u_2(t)b_2(t)dt-\frac{1}{2k}\int_d^x t^{-2l}q(t)b_2(t)dt$$
$$-\frac{a^-k}{2}\int_0^d u_1(t)a_2(t)dt+\frac{a^-}{2k}\int_0^d t^{-2l}q(t)a_2(t)dt$$

$$b_2(x)=\frac{a^-k}{2}\int_0^d u_1(t)dt+\frac{a^-k}{2}\int_0^d u_2(t)dt+\frac{a^-k}{2}\int_0^d u_2(t)a_1(t)dt-\frac{a^+}{2k}\int_0^d t^{-2l}q(t)dt-\frac{a^-k}{2}\int_d^x u_1(t)dt$$
$$-\frac{k}{2}\int_d^x u_1(t)b_1(t)dt-\frac{a^-k}{2}\int_d^x u_2(t)dt-\frac{k}{2}\int_d^x u_2(t)b_1(t)dt+\frac{a^-}{2k}\int_d^x t^{-2l}q(t)dt+\frac{1}{2k}\int_d^x t^{-2l}q(t)b_1(t)dt$$
$$+\frac{a^-k}{2}\int_d^x u_1(t)a_1(t)dt-\frac{a^-}{2k}\int_0^d t^{-2l}q(t)a_1(t)dt$$

Şimdi

1-) $d < x < 2d,\ -x < t < x-2d < 2d-x$

2-) $2d < x,\ -x < t < 2d-x$

3-) $d < x < 2d,\ x-2d < t < 2d-x$

4-) $2d < x,\ -x < t < x-2d$

5-) $2d < x,\ 2d-x < t < x$

6-) $d < x < 2d,\ x-2d < t < x$

bölgelerinde $K_{ij}(x,t),\ (i,j=1,2)$ fonksiyonlarının ifadelerini alıp ardışık yaklaşımlar yöntemini uygularsak;

1-) $d < x < 2d,\ -x < t < x-2d < 2d-x$ aralığı için,

$$K^{(0)}{}_{11}(x,t) = \frac{a^+k}{4}u_1\left(\tfrac{x+t}{2}\right)a_2\left(\tfrac{x+t}{2}\right)+\frac{a^-k}{4}u_1\left(d-\tfrac{x-t}{2}\right)a_2\left(d-\tfrac{x-t}{2}\right)-\frac{a^+}{4k}\left(\tfrac{x+t}{2}\right)^{-2l}q\left(\tfrac{x+t}{2}\right)a_2\left(\tfrac{x+t}{2}\right)$$
$$-\frac{a^-}{4k}\left(d-\tfrac{x-t}{2}\right)^{-2l}q\left(d-\tfrac{x-t}{2}\right)a_2\left(d-\tfrac{x-t}{2}\right)-\frac{a^+k}{4}u_2\left(\tfrac{x+t}{2}\right)a_2\left(\tfrac{x+t}{2}\right)-\frac{a^-k}{4}u_2\left(d-\tfrac{x-t}{2}\right)a_2\left(d-\tfrac{x-t}{2}\right)$$

$$K^{(n)}{}_{11}(x,t) = -\frac{a^+k}{2}\int_{\frac{x-t}{2}}^{d}K_{12}^{(n-1)}(z,t+z-x)u_1(z)dz+\frac{a^+k}{2}\int_{\frac{x+t}{2}}^{d}K_{12}^{(n-1)}(z,t+x-z)u_1(z)dz$$
$$+\frac{a^-k}{2}\int_{d-\frac{x-t}{2}}^{d}K_{12}^{(n-1)}(z,t+2d-x-z)u_1(z)dz-\frac{a^-k}{2}\int_{d-\frac{x+t}{2}}^{d}K_{12}^{(n-1)}(z,t+x+z-2d)u_1(z)dz$$
$$+\frac{a^+k}{2}\int_{\frac{x-t}{2}}^{d}K_{21}^{(n-1)}(z,t+z-x)u_2(z)dz+\frac{a^+k}{2}\int_{\frac{x+t}{2}}^{d}K_{21}^{(n-1)}(z,t+x-z)u_2(z)dz+\frac{a^+}{2}\int_0^d q(s)s^{-2l}\int_{t-x+s}^{t+x-s}K_{11}^{(n-1)}(s,z)dz\,ds$$
$$-\int_0^d q(s)s^{-2l}\int_{t-x-s+2d}^{t+x+s-2d}K_{11}^{(n-1)}(s,z)dz\,ds-\frac{k}{2}\int_{\frac{x-t}{2}}^{d}K_{12}^{(n-1)}(z,t+z-x)u_1(z)dz-\frac{k}{2}\int_d^x K_{12}^{(n-1)}(z,t+z-x)u_1(z)dz$$
$$+\frac{k}{2}\int_d^x K_{12}^{(n-1)}(z,t+x-z)u_1(z)dz+\frac{k}{2}\int_{\frac{x-t}{2}}^{d}K_{21}^{(n-1)}(z,t+z-x)u_2(z)dz+\frac{k}{2}\int_d^x K_{21}^{(n-1)}(z,t+z-x)u_2(z)dz$$
$$+\frac{k}{2}\int_d^x K_{21}^{(n-1)}(z,t+x-z)u_2(z)dz+\frac{k}{4}\int_{2d-x}^{x}u_1\left(\tfrac{x+z}{2}\right)a_2\left(\tfrac{x+z}{2}\right)e^{ikz}dz+\frac{k}{4}\int_x^{2d-x}u_1\left(d-\tfrac{x-z}{2}\right)b_2\left(d-\tfrac{x-z}{2}\right)e^{ikz}dz$$
$$+\frac{1}{2}\int_d^x q(s)s^{-2l}\int_{t-x+s}^{t+x-s}K_{11}^{(n-1)}(s,z)dz\,ds$$

$$K_{12}^{(0)}(x,t) = -\frac{a^+k}{4}u_1\left(\tfrac{x+t}{2}\right)a_1\left(\tfrac{x+t}{2}\right)-\frac{a^-k}{4}u_1\left(d-\tfrac{x-t}{2}\right)a_1\left(d-\tfrac{x-t}{2}\right)+\frac{a^+}{4k}\left(\tfrac{x+t}{2}\right)^{-2l}q\left(\tfrac{x+t}{2}\right)a_1\left(\tfrac{x+t}{2}\right)+\frac{a^-}{4k}\left(d-\tfrac{x-t}{2}\right)^{-2l}q\left(d-\tfrac{x-t}{2}\right)a_1\left(d-\tfrac{x-t}{2}\right)$$
$$-\frac{a^+k}{4}u_1\left(\tfrac{x+t}{2}\right)-\frac{a^-k}{4}u_1\left(d-\tfrac{x-t}{2}\right)+\frac{a^+k}{4}u_2\left(\tfrac{x+t}{2}\right)+\frac{a^-k}{4}u_2\left(d-\tfrac{x-t}{2}\right)+\frac{a^+k}{4}u_2\left(\tfrac{x+t}{2}\right)a_1\left(\tfrac{x+t}{2}\right)+\frac{a^-k}{4}u_2\left(d-\tfrac{x-t}{2}\right)a_1\left(d-\tfrac{x-t}{2}\right)$$
$$+\frac{a^+}{4k}\left(\tfrac{x+t}{2}\right)^{-2l}q\left(\tfrac{x+t}{2}\right)+\frac{a^-}{4k}\left(d-\tfrac{x-t}{2}\right)^{-2l}q\left(d-\tfrac{x-t}{2}\right)$$

$$K_{12}^{(n)} = \frac{a^+k}{2} \int_{\frac{x-t}{2}}^{d} K_{11}^{(n-1)}(z, t+z-x) u_1(z) dz - \frac{a^+k}{2} \int_{\frac{x+t}{2}}^{d} K_{11}^{(n-1)}(z, t+x-z) u_1(z) dz$$

$$-\frac{a^-k}{2} \int_{d-\frac{x-t}{2}}^{d} K_{11}^{(n-1)}(z, t+2d-x-z) u_1(z) dz + \frac{a^-k}{2} \int_{d-\frac{x+t}{2}}^{d} K_{12}^{(n-1)}(z, t+x+z-2d) u_1(z) dz$$

$$+\frac{a^+k}{2} \int_{\frac{x-t}{2}}^{d} K_{22}^{(n-1)}(z, t+z-x) u_2(z) dz + \frac{a^+k}{2} \int_{\frac{x+t}{2}}^{d} K_{22}^{(n-1)}(z, t+x-z) u_2(z) dz$$

$$+\frac{a^-k}{2} \int_{d-\frac{x-t}{2}}^{d} K_{22}^{(n-1)}(z, t+2d-x-z) u_2(z) dz + \frac{a^-k}{2} \int_{d-\frac{x+t}{2}}^{d} K_{22}^{(n-1)}(z, t+x+z-2d) u_2(z) dz$$

$$+\frac{a^+}{2} \int_0^d q(s) s^{-2l} \int_{t-x+s}^{t+x-s} K_{12}^{(n-1)}(s,z) dz\, ds - \int_0^d q(s) s^{-2l} \int_{t-x-s+2d}^{t+x+s-2d} K_{12}^{(n-1)}(s,z) dz\, ds + \frac{k}{2} \int_{\frac{x-t}{2}}^{d} K_{11}^{(n-1)}(z, t+z-x) u_1(z) dz$$

$$+\frac{k}{2} \int_d^x K_{11}^{(n-1)}(z, t+z-x) u_1(z) dz - \frac{k}{2} \int_d^x K_{11}^{(n-1)}(z, t+x-z) u_1(z) dz + \frac{k}{2} \int_d^x K_{12}^{(n-1)}(z, t+x-z) u_1(z) dz$$

$$+\frac{k}{2} \int_{\frac{x-t}{2}}^{x} K_{22}^{(n-1)}(z, t+z-x) u_2(z) dz + \frac{k}{2} \int_d^x K_{22}^{(n-1)}(z, t+z-x) u_2(z) dz + \frac{k}{2} \int_d^x K_{22}^{(n-1)}(z, t+x-z) u_2(z) dz$$

$$+\frac{1}{2} \int_d^x q(s) s^{-2l} \int_{t-x+s}^{t+x-s} K_{12}^{(n-1)}(s,z) dz\, ds$$

$$K^{(0)}{}_{21}(x,t) = -\frac{a^+k}{4} u_1\left(\tfrac{x+t}{2}\right) a_1\left(\tfrac{x+t}{2}\right) - \frac{a^-k}{4} u_1\left(d-\tfrac{x-t}{2}\right) a_1\left(d-\tfrac{x-t}{2}\right) + \frac{a^+}{4k}\left(\tfrac{x+t}{2}\right)^{-2l} q\left(\tfrac{x+t}{2}\right) a_1\left(\tfrac{x+t}{2}\right)$$

$$+\frac{a^-}{4k}\left(d-\tfrac{x-t}{2}\right)^{-2l} q\left(d-\tfrac{x-t}{2}\right) a_1\left(d-\tfrac{x-t}{2}\right) - \frac{a^+k}{4} u_1\left(\tfrac{x+t}{2}\right) + \frac{a^-k}{4} u_1\left(d-\tfrac{x-t}{2}\right) + \frac{a^+k}{4} u_2\left(\tfrac{x+t}{2}\right) - \frac{a^-k}{4} u_2\left(d-\tfrac{x-t}{2}\right)$$

$$+\frac{a^+k}{4} u_2\left(\tfrac{x+t}{2}\right) a_1\left(\tfrac{x+t}{2}\right) - \frac{a^-k}{4} u_2\left(d-\tfrac{x-t}{2}\right) a_1\left(d-\tfrac{x-t}{2}\right) + \frac{a^+}{4k}\left(\tfrac{x+t}{2}\right)^{-2l} q\left(\tfrac{x+t}{2}\right) - \frac{a^-}{4k}\left(d-\tfrac{x-t}{2}\right)^{-2l} q\left(d-\tfrac{x-t}{2}\right)$$

$$K_{21}^{(n)} = -\frac{a^+k}{2} \int_{\frac{x-t}{2}}^{d} K_{11}^{(n-1)}(z, t+z-x) u_1(z) dz - \frac{a^+k}{2} \int_{\frac{x+t}{2}}^{d} K_{11}^{(n-1)}(z, t+x-z) u_1(z) dz$$

$$+\frac{a^-k}{2} \int_{d-\frac{x-t}{2}}^{d} K_{11}^{(n-1)}(z, t+2d-x-z) u_1(z) dz + \frac{a^-k}{2} \int_{d-\frac{x+t}{2}}^{d} K_{12}^{(n-1)}(z, t+x+z-2d) u_1(z) dz$$

$$-\frac{a^+k}{2} \int_{\frac{x-t}{2}}^{d} K_{22}^{(n-1)}(z, t+z-x) u_2(z) dz + \frac{a^+k}{2} \int_{\frac{x+t}{2}}^{d} K_{22}^{(n-1)}(z, t+x-z) u_2(z) dz$$

$$-\frac{a^-k}{2} \int_{d-\frac{x-t}{2}}^{d} K_{22}^{(n-1)}(z, t+2d-x-z) u_2(z) dz + \frac{a^-k}{2} \int_{d-\frac{x+t}{2}}^{d} K_{22}^{(n-1)}(z, t+x+z-2d) u_2(z) dz$$

$$+\frac{a^+}{2k} \int_{\frac{x-t}{2}}^{d} K_{11}^{(n-1)}(z, t+z-x) q(z) z^{-2l} dz + \frac{a^+}{2k} \int_{\frac{x+t}{2}}^{d} K_{11}^{(n-1)}(z, t+x-z) q(z) z^{-2l} dz$$

$$-\frac{a^-}{2k}\int_{d-\frac{x-t}{2}}^{d} K_{11}^{(n-1)}(z,t+2d-x-z)q(z)z^{-2l}dz-\frac{a-}{2k}\int_{\frac{x+t}{2}}^{d} K_{11}^{(n-1)}(z,t+x+z-2d)q(z)z^{-2l}dz$$

$$-\frac{k}{2}\int_{\frac{x-t}{2}}^{d} K_{11}^{(n-1)}(z,t+z-x)u_1(z)dz-\frac{k}{2}\int_{d}^{x} K_{11}^{(n-1)}(z,t+z-x)u_1(z)dz-\frac{k}{2}\int_{d}^{x} K_{11}^{(n-1)}(z,t+x-z)u_1(z)dz-$$

$$\frac{k}{2}\int_{\frac{x-t}{2}}^{x} K_{22}^{(n-1)}(z,t+z-x)u_2(z)dz-\frac{k}{2}\int_{d}^{x} K_{22}^{(n-1)}(z,t+z-x)u_2(z)dz+\frac{k}{2}\int_{d}^{x} K_{22}^{(n-1)}(z,t+x-z)u_2(z)dz$$

$$+\frac{1}{2k}\int_{\frac{x-t}{2}}^{x} K_{11}^{(n-1)}(z,t+z-x)q(z)z^{-2l}dz+\frac{1}{2k}\int_{d}^{x} K_{11}^{(n-1)}(z,t+z-x)q(z)z^{-2l}dz+\frac{1}{2k}\int_{d}^{x} K_{11}^{(n-1)}(z,t+x-z)q(z)z^{-2l}dz$$

$$K^{(0)}{}_{22}(x,t)=-\frac{a^+k}{4}u_1\left(\tfrac{x+t}{2}\right)a_2\left(\tfrac{x+t}{2}\right)+\frac{a^-k}{4}u_1\left(d-\tfrac{x-t}{2}\right)a_2\left(d-\tfrac{x-t}{2}\right)+\frac{a^+}{4k}\left(\tfrac{x+t}{2}\right)^{-2l}q\left(\tfrac{x+t}{2}\right)a_2\left(\tfrac{x+t}{2}\right)$$

$$-\frac{a^-}{4k}\left(d-\tfrac{x-t}{2}\right)^{-2l}q\left(d-\tfrac{x-t}{2}\right)a_2\left(d-\tfrac{x-t}{2}\right)-\frac{a^+k}{4}u_2\left(\tfrac{x+t}{2}\right)a_2\left(\tfrac{x+t}{2}\right)-\frac{a^-k}{4}u_2\left(d-\tfrac{x-t}{2}\right)a_2\left(d-\tfrac{x-t}{2}\right)$$

$$K_{22}^{(n)}=-\frac{a^+k}{2}\int_{\frac{x-t}{2}}^{d} K_{12}^{(n-1)}(z,t+z-x)u_1(z)dz-\frac{a^+k}{2}\int_{\frac{x+t}{2}}^{d} K_{12}^{(n-1)}(z,t+x-z)u_1(z)dz$$

$$+\frac{a^-k}{2}\int_{d-\frac{x-t}{2}}^{d} K_{11}^{(n-1)}(z,t+2d-x-z)u_1(z)dz+\frac{a^-k}{2}\int_{d-\frac{x+t}{2}}^{d} K_{12}^{(n-1)}(z,t+x+z-2d)u_1(z)dz$$

$$+\frac{a^+k}{2}\int_{\frac{x-t}{2}}^{d} K_{21}^{(n-1)}(z,t+z-x)u_2(z)dz-\frac{a^+k}{2}\int_{\frac{x+t}{2}}^{d} K_{21}^{(n-1)}(z,t+x-z)u_2(z)dz$$

$$+\frac{a^-k}{2}\int_{d-\frac{x-t}{2}}^{d} K_{21}^{(n-1)}(z,t+2d-x-z)u_2(z)dz-\frac{a^-k}{2}\int_{d-\frac{x+t}{2}}^{d} K_{21}^{(n-1)}(z,t+x+z-2d)u_2(z)dz$$

$$+\frac{a^+}{2k}\int_{\frac{x-t}{2}}^{d} K_{12}^{(n-1)}(z,t+z-x)q(z)z^{-2l}dz+\frac{a^+}{2k}\int_{\frac{x+t}{2}}^{d} K_{12}^{(n-1)}(z,t+x-z)q(z)z^{-2l}dz$$

$$-\frac{a^-}{2k}\int_{d-\frac{x-t}{2}}^{d} K_{12}^{(n-1)}(z,t+2d-x-z)q(z)z^{-2l}dz-\frac{a^-}{2k}\int_{d-\frac{x+t}{2}}^{d} K_{12}^{(n-1)}(z,t+x+z-2d)q(z)z^{-2l}dz$$

$$-\frac{k}{2}\int_{\frac{x-t}{2}}^{d} K_{12}^{(n-1)}(z,t+z-x)u_1(z)dz-\frac{k}{2}\int_{d}^{x} K_{12}^{(n-1)}(z,t+z-x)u_1(z)dz-\frac{k}{2}\int_{d}^{x} K_{12}^{(n-1)}(z,t+x-z)u_1(z)dz-$$

$$-\frac{k}{2}\int_{d}^{x} K_{12}^{(n-1)}(z,t+x-z)u_1(z)dz+\frac{k}{2}\int_{\frac{x-t}{2}}^{x} K_{21}^{(n-1)}(z,t+z-x)u_2(z)dz-\frac{k}{2}\int_{d}^{x} K_{21}^{(n-1)}(z,t+z-x)u_2(z)dz$$

$$-\frac{k}{2}\int_{\frac{x-t}{2}}^{x} K_{22}^{(n-1)}(z,t+z-x)u_2(z)dz-\frac{k}{2}\int_{d}^{x} K_{22}^{(n-1)}(z,t+z-x)u_2(z)dz+\frac{1}{2k}\int_{\frac{x-t}{2}}^{x} K_{12}^{(n-1)}(z,t+z-x)q(z)z^{-2l}dz$$

$$+\frac{1}{2k}\int_{d}^{x} K_{12}^{(n-1)}(z,t+z-x)q(z)z^{-2l}dz+\frac{1}{2k}\int_{d}^{x} K_{12}^{(n-1)}(z,t+x-z)q(z)z^{-2l}dz$$

integral denklemlerini alırız. Her bir denklemin önce mutlak değerini alır ve sonra $[-x,x]$ aralığında t ye göre integrallersek

$$\int_{-x}^{x}\left|K_{11}^{(0)}(x,t)\right|dt \le \frac{a^{+}|k|}{2}\int_{0}^{x}|u_1(z)a_2(z)|\,dz + \frac{|a^{-}||k|}{2}\int_{d-x}^{x}|u_1(z)a_2(z)|\,dz + \frac{a^{+}}{2|k|}\int_{0}^{x} z^{-2l}|q(z)a_2(z)|\,dz$$
$$+\frac{|a^{-}|}{2|k|}\int_{d-x}^{d} z^{-2l}|q(z)a_2(z)|\,dz + \frac{a^{+}|k|}{2}\int_{0}^{x}|u_2(z)a_2(z)|\,dz + \frac{|a^{-}||k|}{2}\int_{d-x}^{x}|u_2(z)a_2(z)|\,dz$$

$$\int_{-x}^{x}\left|K_{12}^{(0)}(x,t)\right|dt \le \frac{a^{+}|k|}{2}\int_{0}^{x}|u_1(z)a_1(z)|\,dz + \frac{|a^{-}||k|}{2}\int_{d-x}^{d}|u_1(z)a_1(z)|\,dz + \frac{a^{+}}{2|k|}\int_{0}^{x} z^{-2l}|q(z)a_{21}(z)|\,dz$$
$$+\frac{|a^{-}|}{2|k|}\int_{d-x}^{d} z^{-2l}|q(z)a_1(z)|\,dz + \frac{a^{+}|k|}{2}\int_{0}^{x}|u_1(z)|\,dz + \frac{|a^{-}||k|}{2}\int_{d-x}^{d}|u_1(z)|\,dz + \frac{a^{+}|k|}{2}\int_{0}^{x}|u_2(z)|\,dz + \frac{|a^{-}||k|}{2}\int_{d-x}^{d}|u_2(z)|\,dz$$
$$+\frac{a^{+}|k|}{2}\int_{0}^{x}|u_2(z)a_1(z)|\,dz + \frac{a^{-}k}{2}\int_{d-x}^{d}|u_2(z)a_1(z)|\,dz + \frac{a^{+}}{2|k|}\int_{0}^{x} z^{-2l}|q(z)|\,dz + \frac{|a^{-}|}{2|k|}\int_{d-x}^{d} z^{-2l}|q(z)|\,dz$$

$$\int_{-x}^{x}\left|K_{21}^{(0)}(x,t)\right|dt \le \frac{a^{+}|k|}{2}\int_{0}^{x}|u_1(z)a_1(z)|\,dz + \frac{|a^{-}||k|}{2}\int_{d-x}^{d}|u_1(z)a_1(z)|\,dz + \frac{a^{+}}{2|k|}\int_{0}^{x} z^{-2l}|q(z)a_1(z)|\,dz$$
$$+\frac{|a^{-}|}{2|k|}\int_{d-x}^{d} z^{-2l}|q(z)a_1(z)|\,dz + \frac{a^{+}|k|}{2}\int_{0}^{x}|u_1(z)|\,dz + \frac{|a^{-}|^{-}|k|}{2}\int_{d-x}^{d}|u_1(z)|\,dz + \frac{a^{+}|k|}{2}\int_{0}^{x}|u_2(z)|\,dz + \frac{a^{-}|k|}{2}\int_{d-x}^{d}|u_2(z)|\,dz$$
$$+\frac{a^{+}|k|}{2}\int_{0}^{x}|u_2(z)a_1(z)|\,dz + \frac{a^{-}k}{2}\int_{0}^{x}|u_2(z)a_1(z)|\,dz + \frac{a^{+}}{2|k|}\int_{0}^{x} z^{-2l}|q(z)|\,dz - \frac{|a^{-}|}{2|k|}\int_{d-x}^{d} z^{-2l}|q(z)|\,dz$$

$$\int_{-x}^{x}K_{22}^{(0)}(x,t)dt \le \frac{a^{+}|k|}{2}\int_{0}^{x}u_1(z)a_2(z)dz + \frac{|a^{-}||k|}{2}\int_{d-x}^{x}u_1(z)a_2(z)dz + \frac{a^{+}}{2|k|}\int_{0}^{x} z^{-2l}q(z)a_2(z)dz$$
$$+\frac{|a^{-}|}{2|k|}\int_{d-x}^{d} z^{-2l}q(z)a_2(z)dz + \frac{a^{+}|k|}{2}\int_{0}^{x}u_2(z)a_2(z)dz + \frac{|a^{-}||k|}{2}\int_{d-x}^{x}u_2(z)a_2(z)dz$$

olur. $a_1(x)$ ve $a_2(x)$ fonksiyonları mutlak sürekli fonksiyonlar ve dolayısıyla sınırlı olduklarından $|a_1(x)|,\ |a_2(x)| < M$ olacak şekilde $M>0$ sayısı var olduğundan

$$\int_{-x}^{x}\left|K_{11}^{(0)}(x,t)\right|dt \le Mk(a^{+}+|a^{-}|)\int_{0}^{x}|u_1(t)|+|u_2(t)|+\frac{M}{2|k|}(a^{+}+|a^{-}|)\int_{0}^{x}\left|z^{-2l}q(z)\right|dz$$
$$\int_{-x}^{x}\left|K_{12}^{(0)}(x,t)\right|dt \le (M+1)k(a^{+}+|a^{-}|)\int_{0}^{x}|u_1(t)|+|u_2(t)|+\frac{M+1}{|k|}(a^{+}+|a^{-}|)\int_{0}^{x}\left|z^{-2l}q(z)\right|dz$$
$$\int_{-x}^{x}\left|K_{21}^{(0)}(x,t)\right|dt \le (M+1)k(a^{+}+|a^{-}|)\int_{0}^{x}|u_1(t)|+|u_2(t)|+\frac{M+1}{|k|}(a^{+}+|a^{-}|)\int_{0}^{x}\left|z^{-2l}q(z)\right|dz$$
$$\int_{-x}^{x}\left|K_{22}^{(0)}(x,t)\right|dt \le Mk(a^{+}+|a^{-}|)\int_{0}^{x}|u_1(t)|+|u_2(t)|+\frac{M}{2|k|}(a^{+}+|a^{-}|)\int_{0}^{x}\left|z^{-2l}q(z)\right|dz$$

eşitsizliklerini elde ederiz. $\max((M+1)k(a^+ + |a^-|),\ \frac{M}{2k}(a^+ + |a^-|) = M_1$ olarak alırsak

$$\int_{-x}^{x} |K_{ij}^{(0)}(x,t)|dt \le M_1 \int_0^x |u_1(t)| + |u_2(t)| + t^{-2l}|q(t)|dt = M_1 s_1(x)$$

olur. Burada $s_1(x) = \int_0^x |u_1(t)| + |u_2(t)| + t^{-2l}|q(t)|dt,\ i,j = 1,2.$ Ayrıca

$$\int_{-x}^{x} |K_{11}^{(n)}(x,t)|dt \le \frac{a^+|k|}{2}\int_0^d |u_1(z)| \int_{-z}^{z} |K_{12}^{(n-1)}(z,s)|dsdz + \frac{a^+|k|}{2}\int_0^d |u_1(z)| \int_{-z}^{z} |K_{12}^{(n-1)}(z,s)|dsdz$$
$$+\frac{|a^-||k|}{2}\int_{d-x}^{x} |u_1(z)| \int_{2d-2x-z}^{z} |K_{12}^{(n-1)}(z,s)|dsdz + \frac{|a^-||k|}{2}\int_{d-x}^{x} |u_1(z)| \int_{-z}^{2x+z-2d} |K_{12}^{(n-1)}(z,s)|dsdz$$
$$+\frac{a^+k}{2}\int_0^d |u_2(z)| \int_{-z}^{z} |K_{21}^{(n-1)}(z,s)|dsdz + \frac{a^+|k|}{2}\int_0^d |u_2(z)| \int_{-z}^{z} |K_{21}^{(n-1)}(z,s)|dsdz$$
$$+\frac{|a^-||k|}{2}\int_{d-x}^{x} |u_2(z)| \int_{2d-2x-z}^{z} |K_{21}^{(n-1)}(z,s)|dsdz + \frac{|a^-||k|}{2}\int_{d-x}^{x} |u_2(z)| \int_{-z}^{2x+z-2d} |K_{21}^{(n-1)}(z,s)|dsdz$$
$$+\frac{a^+}{2}\int_0^d |q(s)| s^{-2l}\left\{\int_{s-2x}^{2x-s}\int_{z-x+s}^{z+x-s} |K_{11}^{(n-1)}(z,s)|dtdz\right\}ds + \frac{|a^-|}{2}\int_0^d |q(s)| s^{-2l}\left\{\int_{2d-2x-s}^{2x+s-2d}\int_{z-x-s+2d}^{z+x+s-2d} |K_{11}^{(n-1)}(z,s)|dtdz\right\}ds$$
$$+\frac{|k|}{2}\int_0^x |u_1(z)| \int_{-z}^{z} |K_{12}^{(n-1)}(z,s)|dsdz + \frac{|k|}{2}\int_d^x |u_1(z)| \int_{z-2x}^{z} |K_{12}^{(n-1)}(z,s)|dsdz + \frac{|k|}{2}\int_d^x |u_1(z)| \int_{-z}^{2x-z} |K_{12}^{(n-1)}(z,s)|dsdz$$
$$+\frac{|k|}{2}\int_0^x u_2(z) \int_{-z}^{z} |K_{21}^{(n-1)}(z,s)|dsdz + \frac{|k|}{2}\int_d^x u_2(z) \int_{z-2x}^{z} |K_{21}^{(n-1)}(z,s)|dsdz + \frac{|k|}{2}\int_d^x |u_2(z|) \int_{-z}^{2x-z} |K_{21}^{(n-1)}(z,s)|dsdz$$
$$+\frac{1}{2}\int_d^x |q(s)| s^{-2l}\left\{\int_{s-2x}^{2x-s}\int_{z-x+s}^{z+x-s} |K_{11}^{(n-1)}(z,s)|dtdz\right\}ds$$

$$\int_{-x}^{x} |K_{12}^{(n)}(x,t)|dt \le \frac{a^+|k|}{2}\int_0^d |u_1(z)| \int_{-z}^{z} |K_{11}^{(n-1)}(z,s)|dsdz + \frac{a^+|k|}{2}\int_0^d |u_1(z)| \int_{-z}^{z} |K_{11}^{(n-1)}(z,s)|dsdz$$
$$+\frac{|a^-||k|}{2}\int_{d-x}^{x} |u_1(z)| \int_{2d-2x-z}^{z} |K_{11}^{(n-1)}(z,s)|dsdz + \frac{a^-|k|}{2}\int_{d-x}^{x} |u_1(z)| \int_{-z}^{2x+z-2d} |K_{11}^{(n-1)}(z,s)|dsdz$$
$$+\frac{a^+|k|}{2}\int_0^d |u_2(z)| \int_{-z}^{z} |K_{22}^{(n-1)}(z,s)|dsdz + \frac{a^+k}{2}\int_0^d |u_2(z)| \int_{-z}^{z} |K_{22}^{(n-1)}(z,s)|dsdz$$
$$+\frac{|a^-||k|}{2}\int_{d-x}^{x} |u_2(z|) \int_{2d-2x-z}^{z} |K_{22}^{(n-1)}(z,s)|dsdz + \frac{|a^-||k|}{2}\int_{d-x}^{x} |u_2(z|) \int_{-z}^{2x+z-2d} |K_{22}^{(n-1)}(z,s)|dsdz$$
$$+\frac{a^+}{2}\int_0^d |q(s)| s^{-2l}\left\{\int_{s-2x}^{2x-s}\int_{z-x+s}^{z+x-s} |K_{12}^{(n-1)}(z,s)|dtdz\right\}ds + \frac{|a^-|}{2}\int_0^d |q(s)| s^{-2l}\left\{\int_{2d-2x-s}^{2x+s-2d}\int_{z-x-s+2d}^{z+x+s-2d} |K_{12}^{(n-1)}(z,s)|dtdz\right\}ds$$

$$+\frac{|k|}{2}\int_0^x |u_1(z)|\int_{-z}^{z}\left|K_{11}^{(n-1)}(z,s)\right|dsdz+\frac{|k|}{2}\int_d^x |u_1(z)|\int_{z-2x}^{z}\left|K_{11}^{(n-1)}(z,s)\right|dsdz+\frac{|k|}{2}\int_d^x |u_1(z)|\int_{-z}^{2x-z}\left|K_{11}^{(n-1)}(z,s)\right|dsdz$$

$$+\frac{|k|}{2}\int_d^x |u_1(z)|\int_{-z}^{2x-z}\left|K_{12}^{(n-1)}(z,s)\right|dsdz+\frac{|k|}{2}\int_0^x |u_2(z)|\int_{-z}^{z}\left|K_{22}^{(n-1)}(z,s)\right|dsdz+\frac{|k|}{2}\int_d^x |u_2(z)|\int_{z-2x}^{z}\left|K_{22}^{(n-1)}(z,s)\right|dsdz$$

$$+\frac{|k|}{2}\int_d^x |u_2(z)|\int_{-z}^{2x-z}\left|K_{22}^{(n-1)}(z,s)\right|dsdz+\frac{1}{2}\int_d^x |q(s)|\,s^{-2l}\left\{\int_{s-2x}^{2x-s}\int_{z-x+s}^{z+x-s}\left|K_{11}^{(n-1)}(z,s)\right|dtdz\right\}ds$$

$$\int_{-x}^{x}\left|K_{21}^{(n)}(x,t)\right|dt\le\frac{a^+|k|}{2}\int_0^d |u_1(z|)\int_{-z}^{z}\left|K_{11}^{(n-1)}(z,s)\right|dsdz+\frac{a^+|k|}{2}\int_0^d |u_1(z)|\left|\int_{-z}^{z}K_{11}^{(n-1)}(z,s)\right|dsdz$$

$$+\frac{|a^-||k|}{2}\int_{d-x}^{x}|u_1(z)|\int_{2d-2x-z}^{z}\left|K_{11}^{(n-1)}(z,s)\right|dsdz+\frac{|a^-||k|}{2}\int_{d-x}^{x}|u_1(z)|\int_{-z}^{2x+z-2d}\left|K_{11}^{(n-1)}(z,s)\right|dsdz$$

$$+\frac{a^+|k|}{2}\int_0^d |u_2(z)|\int_{-z}^{z}\left|K_{22}^{(n-1)}(z,s)\right|dsdz+\frac{a^+|k|}{2}\int_0^d |u_2(z)|\int_{-z}^{z}\left|K_{22}^{(n-1)}(z,s)\right|dsdz$$

$$+\frac{|a^-||k|}{2}\int_{d-x}^{x}|u_2(z)|\int_{2d-2x-z}^{z}\left|K_{22}^{(n-1)}(z,s)\right|dsdz+\frac{|a^-||k|}{2}\int_{d-x}^{x}|u_2(z)|\int_{-z}^{2x+z-2d}\left|K_{22}^{(n-1)}(z,s)\right|dsdz$$

$$+\frac{a^+}{2|k|}\int_0^d |q(z)|z^{-2l}\int_{-z}^{z}\left|K_{11}^{(n-1)}(z,s)\right|dsdz+\frac{a^+}{2|k|}\int_0^d |q(z)|z^{-2l}\int_{-z}^{z}\left|K_{11}^{(n-1)}(z,s)\right|dsdz$$

$$+\frac{|a^-|}{2|k|}\int_{d-x}^{d}|q(z)|z^{-2l}\int_{2d-2x-z}^{z}\left|K_{11}^{(n-1)}(z,s)\right|dsdz+\frac{|a^-|}{2|k|}\int_{d-x}^{d}|q(z)|z^{-2l}\int_{-z}^{2x+z-2d}\left|K_{11}^{(n-1)}(z,s)\right|dsdz$$

$$+\frac{|k|}{2}\int_0^x |u_1(z)|\int_{-z}^{z}\left|K_{11}^{(n-1)}(z,s)\right|dsdz+\frac{|k|}{2}\int_d^x u_1(z)\int_{z-2x}^{z}\left|K_{11}^{(n-1)}(z,s)\right|dsdz+\frac{|k|}{2}\int_d^x |u_1(z)|\int_{-z}^{2x-z}\left|K_{11}^{(n-1)}(z,s)\right|dsdz$$

$$+\frac{|k|}{2}\int_0^x |u_2(z)|\int_{-z}^{z}\left|K_{22}^{(n-1)}(z,s)\right|dsdz+\frac{|k|}{2}\int_d^x |u_2(z)|\int_{z-2x}^{z}\left|K_{22}^{(n-1)}(z,s)\right|dsdz+\frac{|k|}{2}\int_d^x |u_2(z)|\int_{-z}^{2x-z}\left|K_{22}^{(n-1)}(z,s)\right|dsdz$$

$$+\frac{1}{2|k|}\int_0^d |q(z)|z^{-2l}\int_{-z}^{z}\left|K_{11}^{(n-1)}(z,s)\right|dsdz+\frac{1}{2|k|}\int_d^x |q(z)|z^{-2l}\int_{z-2x}^{z}\left|K_{11}^{(n-1)}(z,s)\right|dsdz$$

$$+\frac{1}{2|k|}\int_d^x |q(z)|z^{-2l}\int_{-z}^{2x-z}\left|K_{11}^{(n-1)}(z,s)\right|dsdz$$

$$\int_{-x}^{x}\left|K_{22}^{(n)}(x,t)\right|dt\le\frac{a^+|k|}{2}\int_0^d |u_1(z)|\int_{-z}^{z}\left|K_{12}^{(n-1)}(z,s)\right|dsdz+\frac{a^+|k|}{2}\int_0^d |u_1(z)|\int_{-z}^{z}\left|K_{12}^{(n-1)}(z,s)\right|dsdz$$

$$+\frac{|a^-||k|}{2}\int_{d-x}^{x}|u_1(z)|\int_{2d-2x-z}^{z}\left|K_{12}^{(n-1)}(z,s)\right|dsdz+\frac{|a^-||k|}{2}\int_{d-x}^{x}|u_1(z)|\int_{-z}^{2x+z-2d}\left|K_{12}^{(n-1)}(z,s)\right|dsdz$$

$$+\frac{a^+|k|}{2}\int_0^d u_2(z)\int_{-z}^{z}\left|K_{21}^{(n-1)}(z,s)\right|dsdz+\frac{a^+|k|}{2}\int_0^d u_2(z)\int_{-z}^{z}\left|K_{21}^{(n-1)}\right|(z,s)dsdz$$

$$+\frac{|a^-||k|}{2}\int_{d-x}^{x}u_2(z)\int_{2d-2x-z}^{z}\left|K_{21}^{(n-1)}(z,s)\right|dsdz+\frac{|a^-||k|}{2}\int_{d-x}^{x}|u_2(z)|\int_{-z}^{2x+z-2d}\left|K_{21}^{(n-1)}(z,s)\right|dsdz$$

$$+\frac{a^+}{2|k|}\int_{0}^{d}|q(z)|z^{-2l}\int_{-z}^{z}\left|K_{12}^{(n-1)}(z,s)\right|dsdz+\frac{a^+}{2|k|}\int_{0}^{d}|q(z)|z^{-2l}\int_{-z}^{z}\left|K_{12}^{(n-1)}(z,s)\right|dsdz$$

$$+\frac{|a^-|}{2|k|}\int_{d-x}^{d}|q(z)|z^{-2l}\int_{2d-2x-z}^{z}\left|K_{12}^{(n-1)}(z,s)\right|dsdz+\frac{|a^-|}{2|k|}\int_{d-x}^{d}|q(z)|z^{-2l}\int_{-z}^{2x+z-2d}\left|K_{12}^{(n-1)}(z,s)\right|dsdz$$

$$+\frac{|k|}{2}\int_{0}^{x}|u_1(z)|\int_{-z}^{z}\left|K_{12}^{(n-1)}(z,s)\right|dsdz+\frac{|k|}{2}\int_{d}^{x}|u_1(z)|\int_{z-2x}^{z}\left|K_{12}^{(n-1)}(z,s)\right|dsdz$$

$$+\frac{|k|}{2}\int_{d}^{x}|u_1(z)|\int_{-z}^{2x-z}\left|K_{12}^{(n-1)}(z,s)\right|dsdz+\frac{|k|}{2}\int_{0}^{x}|u_2(z)|\int_{-z}^{z}\left|K_{21}^{(n-1)}(z,s)\right|dsdz+\frac{|k|}{2}\int_{d}^{x}|u_2(z)|\left|\int_{z-2x}^{z}K_{21}^{(n-1)}(z,s)\right|dsdz$$

$$+\frac{|k|}{2}\int_{d}^{x}|u_2(z)|\int_{-z}^{2x-z}\left|K_{21}^{(n-1)}(z,s)\right|dsdz+\frac{k}{2}\int_{0}^{x}|u_2(z)|\int_{-z}^{2x-z}\left|K_{22}^{(n-1)}(z,s)\right|dsdz+\frac{|k|}{2}\int_{d}^{x}|u_2(z)|\int_{z-2x}^{z}\left|K_{22}^{(n-1)}(z,s)\right|dsdz$$

$$+\frac{1}{2|k|}\int_{0}^{d}|q(z)|z^{-2l}\int_{-z}^{z}\left|K_{12}^{(n-1)}(z,s)\right|dsdz+\frac{1}{2|k|}\int_{d}^{x}|q(z)|z^{-2l}\int_{z-2x}^{z}\left|K_{12}^{(n-1)}(z,s)\right|dsdz$$

$$+\frac{1}{2|k|}\int_{d}^{x}|q(z)|z^{-2l}\int_{-z}^{2x-z}\left|K_{12}^{(n-1)}(z,s)\right|dsdz$$

eşitsizliklerini kullanırsak,

$$\int_{-x}^{x}\left|K_{11}^{(1)}(x,t)\right|dt\le\frac{a^+|k|}{2}M_1\frac{s_1^2(x)}{2!}+\frac{a^+|k|}{2}M_1\frac{s_1^2(x)}{2!}+\frac{|a^-||k|}{2}M_1\frac{s_1^2(x)}{2!}+\frac{|a^-||k|}{2}M_1\frac{s_1^2(x)}{2!}+\frac{a^+|k|}{2}M_1\frac{s_1^2(x)}{2!}$$

$$+\frac{a^+|k|}{2}M_1\frac{s_1^2(x)}{2!}+\frac{|a^-||k|}{2}M_1\frac{s_1^2(x)}{2!}+\frac{|a^-||k|}{2}M_1\frac{s_1^2(x)}{2!}+\frac{a^+}{2}M_1c_1\frac{s_1^2(x)}{2!}+\frac{|a^-|}{2}M_1c_2\frac{s_1^2(x)}{2!}+\frac{|k|}{2}M_1\frac{s_1^2(x)}{2!}$$

$$+\frac{|k|}{2}M_1\frac{s_1^2(x)}{2!}+\frac{|k|}{2}M_1\frac{s_1^2(x)}{2!}+\frac{|k|}{2}M_1\frac{s_1^2(x)}{2!}+\frac{|k|}{2}M_1\frac{s_1^2(x)}{2!}+\frac{|k|}{2}M_1\frac{s_1^2(x)}{2!}+\frac{1}{2}M_1c_1\frac{s_1^2(x)}{2!}=$$

$$=\left(2a^+|k|+2a^-|k|+3k+\frac{1+a^+}{2}c_1+\frac{|a^-|}{2}c_2\right)M_1\frac{s_1^2(x)}{2!}$$

$$\int_{-x}^{x}\left|K_{12}^{(1)}(x,t)\right|dt\le\frac{a^+|k|}{2}M_1\frac{s_1^2(x)}{2!}+\frac{a^+|k|}{2}M_1\frac{s_1^2(x)}{2!}+\frac{|a^-||k|}{2}M_1\frac{s_1^2(x)}{2!}+\frac{a^-|k|}{2}M_1\frac{s_1^2(x)}{2!}+\frac{a^+|k|}{2}M_1\frac{s_1^2(x)}{2!}$$

$$+\frac{a^+|k|}{2}M_1\frac{s_1^2(x)}{2!}+\frac{|a^-||k|}{2}M_1\frac{s_1^2(x)}{2!}+\frac{|a^-||k|}{2}M_1\frac{s_1^2(x)}{2!}+\frac{a^+}{2}M_1c_1\frac{s_1^2(x)}{2!}+\frac{|a^-|}{2}M_1c_2\frac{s_1^2(x)}{2!}+\frac{|k|}{2}M_1\frac{s_1^2(x)}{2!}$$

$$+\frac{|k|}{2}M_1\frac{s_1^2(x)}{2!}+\frac{|k|}{2}M_1\frac{s_1^2(x)}{2!}+\frac{|k|}{2}M_1\frac{s_1^2(x)}{2!}+\frac{|k|}{2}M_1\frac{s_1^2(x)}{2!}+\frac{|k|}{2}M_1\frac{s_1^2(x)}{2!}+\frac{|k|}{2}M_1\frac{s_1^2(x)}{2!}+\frac{1}{2}M_1c_1\frac{s_1^2(x)}{2!}$$

$$=\left(2a^+|k|+\frac{7|k|}{2}+\frac{1+a^+}{2}c_1+\frac{|a^-|}{2}c_2\right)M_1\frac{s_1^2(x)}{2!}$$

$$\int_{-x}^{x}\left|K_{21}^{(1)}(x,t)\right|dt \le \frac{a^+|k|}{2}M_1\frac{s_1^2(x)}{2!}+\frac{a^+|k|}{2}M_1\frac{s_1^2(x)}{2!}+\frac{|a^-||k|}{2}M_1\frac{s_1^2(x)}{2!}+\frac{|a^-||k|}{2}M_1\frac{s_1^2(x)}{2!}+\frac{a^+|k|}{2}M_1\frac{s_1^2(x)}{2!}$$

$$+\frac{a^+|k|}{2}M_1\frac{s_1^2(x)}{2!}+\frac{|a^-||k|}{2}M_1\frac{s_1^2(x)}{2!}+\frac{|a^-||k|}{2}M_1\frac{s_1^2(x)}{2!}+\frac{a^+}{2|k|}M_1\frac{s_1^2(x)}{2!}+\frac{a^+}{2|k|}M_1\frac{s_1^2(x)}{2!}+\frac{|a^-|}{2|k|}M_1\frac{s_1^2(x)}{2!}$$

$$+\frac{|a^-|}{2|k|}M_1\frac{s_1^2(x)}{2!}+\frac{|k|}{2}M_1\frac{s_1^2(x)}{2!}+\frac{|k|}{2}M_1\frac{s_1^2(x)}{2!}+\frac{|k|}{2}M_1\frac{s_1^2(x)}{2!}+\frac{|k|}{2}\int_0^x|u_2(z)|\int_{-z}^{z}\left|K_{22}^{(n-1)}(z,s)\right|dsdz$$

$$+\frac{|k|}{2}M_1\frac{s_1^2(x)}{2!}+\frac{|k|}{2}M_1\frac{s_1^2(x)}{2!}+\frac{1}{2|k|}M_1\frac{s_1^2(x)}{2!}+\frac{1}{2|k|}M_1\frac{s_1^2(x)}{2!}+\frac{1}{2|k|}M_1\frac{s_1^2(x)}{2!}$$

$$=\left(2a^+|k|+2|a^-||k|+\frac{a^+}{|k|}+\frac{|a^-|}{|k|}+3|k|+\frac{3}{2|k|}\right)M_1\frac{s_1^2(x)}{2!}$$

$$\int_{-x}^{x}\left|K_{22}^{(1)}(x,t)\right|dt \le \frac{a^+|k|}{2}M_1\frac{s_1^2(x)}{2!}+\frac{a^+|k|}{2}M_1\frac{s_1^2(x)}{2!}+\frac{|a^-||k|}{2}M_1\frac{s_1^2(x)}{2!}+\frac{|a^-||k|}{2}M_1\frac{s_1^2(x)}{2!}+\frac{a^+|k|}{2}M_1\frac{s_1^2(x)}{2!}$$

$$+\frac{a^+|k|}{2}M_1\frac{s_1^2(x)}{2!}+\frac{|a^-||k|}{2}M_1\frac{s_1^2(x)}{2!}+\frac{|a^-||k|}{2}M_1\frac{s_1^2(x)}{2!}+\frac{a^+}{2|k|}M_1\frac{s_1^2(x)}{2!}+\frac{a^+}{2|k|}M_1\frac{s_1^2(x)}{2!}+\frac{|a^-|}{2|k|}M_1\frac{s_1^2(x)}{2!}$$

$$+\frac{|a^-|}{2|k|}M_1\frac{s_1^2(x)}{2!}+\frac{|k|}{2}M_1\frac{s_1^2(x)}{2!}+\frac{|k|}{2}M_1\frac{s_1^2(x)}{2!}+\frac{|k|}{2}M_1\frac{s_1^2(x)}{2!}+\frac{|k|}{2}M_1\frac{s_1^2(x)}{2!}+\frac{|k|}{2}M_1\frac{s_1^2(x)}{2!}+\frac{|k|}{2}M_1\frac{s_1^2(x)}{2!}$$

$$+\frac{|k|}{2}M_1\frac{s_1^2(x)}{2!}+\frac{|k|}{2}M_1\frac{s_1^2(x)}{2!}+\frac{1}{2|k|}M_1\frac{s_1^2(x)}{2!}++\frac{1}{2|k|}M_1\frac{s_1^2(x)}{2!}+\frac{1}{2|k|}M_1\frac{s_1^2(x)}{2!}$$

$$=\left(2a^+|k|+2|a^-||k|+\frac{a^+}{|k|}+\frac{|a^-|}{|k|}4k+\frac{3}{2|k|}\right)M_1\frac{s_1^2(x)}{2!}$$

$$\left(2a^+|k|+2a^-|k|+4|k|+\frac{1+a^+}{2}c_1+\frac{|a^-|}{2}c_2+\frac{a^+}{|k|}+\frac{|a^-|}{|k|}+\frac{3}{2|k|}\right)M_1=C$$ alırsak

$\int_{-x}^{x}\left|K_{ij}^{(1)}(x,t)\right|dt \le C^2\frac{s_1^2(x)}{2!}$ eşitsizliğini, aynı şekilde $n=2$ için $\int_{-x}^{x}\left|K_{ij}^{(2)}(x,t)\right|dt \le C^3\frac{s_1^3(x)}{3!}$ eşitsizliğini elde ederiz. Tümevarım yöntemini kullanırsak $\int_{-x}^{x}\left|K_{ij}^{(n)}(x,t)\right|dt \le C^{n+1}\frac{s_1^{n+1}(x)}{(n+1)!}$ eşitsizliğinin geçerli olduğunu alırız. Aynı işlemler diğer bölgeler içinde yapılırsa bu eşitsizlikler kolayca alınabilir. Bu eşitsizliklerden

$\sum_{n=0}^{\infty}\int_{-x}^{x}\left|K_{ij}^{(n)}(x,t)\right|dt$ serisinin $L_1(0,p)$ uzayında düzgün yakınsak olduğu açıktır ve bu serinin toplamı olan $K_{ij}(x,t)\in L_1(0,p)$ fonksiyonu

$$\sum_{n=0}^{\infty}\int_{-x}^{x}\left|K_{ij}^{(n)}(x,t)\right|dt\le e^{cs_1(x)}-1$$

eşitsizliğini sağlar. Bu durumda aşağıdaki teoremi ispatlamış olduk.

Teorem : $\int_{0}^{p}\left|q(t)\right|t^{-2l}dt<\infty$ olsun. (4) diferansiyel denklemler sisteminin $\begin{pmatrix} y_1 \\ y_3 \end{pmatrix}(0)=\begin{pmatrix} 1 \\ i \end{pmatrix}$ başlangıç koşullarını ve (6) süreksizlik koşulunu sağlayan her bir çözümü

$$\begin{pmatrix} y_1 \\ y_3 \end{pmatrix}=\begin{pmatrix} y_{10}+a(x)e^{ikx}+b(x)e^{ik(2d-x)}+\int_{-x}^{x}K_{11}(x,t)e^{ikt}dt+i\int_{-x}^{x}K_{12}(x,t)e^{ikt}dt \\ y_{30}+ia(x)e^{ikx}-ib(x)e^{ik(2d-x)}+\int_{-x}^{x}K_{21}(x,t)e^{ikt}dt+i\int_{-x}^{x}K_{22}(x,t)e^{ikt}dt \end{pmatrix}$$

şeklinde gösterime sahiptir. Ayrıca $s_1(x)=\int_{0}^{x}\left|u_1(t)\right|+\left|u_2(t)\right|+t^{-2l}\left|q(t)\right|dt,\ i,j=1,2.$ olmak üzere

$$\sum_{n=0}^{\infty}\int_{-x}^{x}\left|K_{ij}^{(n)}(x,t)\right|dt\le e^{cs_1(x)}-1$$

eşitsizliği sağlanır. Burada $a(x),\ b(x)\in AC(0,p]$, $\begin{pmatrix} y_{10} \\ y_{30} \end{pmatrix}=\begin{pmatrix} a^{+}e^{ikx}+a^{-}e^{ik(2d-x)} \\ ia^{+}e^{ikx}-ia^{-}e^{ik(2d-x)} \end{pmatrix}$,

$$\max\left(\frac{M+2}{2}\left|k\right|(a^{+}+\left|a^{-}\right|),\frac{1}{2\left|k\right|}(a^{+}+\left|a^{-}\right|)\right)\left(2a^{+}\left|k\right|+2\left|a^{-}\right|\left|k\right|+4\left|k\right|+\frac{1+a^{+}}{2}c_1\right.$$
$$\left.+\frac{\left|a^{-}\right|}{2}c_2+\frac{a^{+}}{\left|k\right|}+\frac{\left|a^{-}\right|}{\left|k\right|}+\frac{3}{2\left|k\right|}\right)=C$$

şeklindedir.

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