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About System of Dirac Differential Equation Which has Discontinıity Point in Semi Axis

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Abstract: In article, existence of transformation operators was proved for a class of Dirac operators which have discontinuity conditions inside of some properties of kernel of this transformation operator was investigated.

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Key words: Dirac operators, transformation operator, discontinuity

Yarı Eksende Süreksizlik Noktalarına Sahip Dirac Diferansiyel Denklemler Hakkında

Özet: Bu çalışmada, aralıkta süreksizlik koşullarına sahip Dirac Operatörlerin bir sınıfı için çevirme operatörünün varlığı ispatlandı ve bu operatörün çekirdeğinin bazı özellikleri incdelendi.

Anahtar Kelimeler: Dirac operatör, çevirme operatörü, süreksizlik

1.Introduction

For solving of the inverse problems for Dirac differential operators as regular as singular the transformation operators have a special place.

The main aim of the present paper is to prove the solution of boundary value problem L which has the type of transformation operator. By using this expression direct and inverse problems of spectral theory can be investigated for Dirac operators. Analogous problem [14] have been investigated in finite interval.

Boundary value problems with discontinuity conditions arise in different branches of mathematics, mechanics, radio, electronics, geophysics and other fields of natural science and technology. For example, discontinuous conditions inside an interval are connected with discontinuous or nonsmooth properties of media ([1], [2]). Inverse problem of this type are connected with the investigation of discontinuous solutions of some nonlinear equations in mathematical physics.

For the classical Sturm-Liouville operators, Schrödinger equation and hyperbolic equations, direct and inverse problems are studied fairly completely (See [3], [5] and references there in).

For Dirac differential equation, direct and inverse problems have been investigated enough. (See [3], [4], [6-8], [14] and references there in). The presence of discontinuity conditions inside an interval introduces qualitative changes in the investigation of such problems. Some aspect of direct and inverse problems for differential operators with discontinuity conditions were studied in [9-14].

Let's get system of Dirac differential equations with canonical form in semi axis,

$$B\frac{dy}{dx}+\Omega(x)y=\lambda y, \qquad 0<\mathrm{x}<\infty \tag{1.1}$$

where $B=\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}$, $\Omega(x)=\begin{pmatrix}p(x) & q(x)\\ q(x) & -p(x)\end{pmatrix}$, $y(x)=\begin{pmatrix}y_1(x)\\ y_2(x)\end{pmatrix}$, $p(x), q(x)\in L_2(0,\infty)$

and λ is a parameter.

In this study, an expression of boundary value problem generated by boundary conditions

$$y(0)=0 \tag{1.2}$$

and

$$y(a-0)=Ay(a+0) \tag{1.3}$$

have been given, where $0<a<+\infty$, $A=\begin{pmatrix}\alpha & 0\\ 0 & 1/\alpha\end{pmatrix}$, $\alpha\neq 1$ is real number.

2. Representation of Solution

Let $Y_0(0,\lambda)$ be a solution of matrice equation (1.1) corresponding to the case of $\Omega(x)\equiv 0$ satisfying $Y_0(0,\lambda)=I$ (I unite matrice) and discontinuity conditions (1.3).

In this case, for the function $Y_0(x,\lambda)$, it is obvious that

$$Y_0(x,\lambda)=\begin{cases} e^{-\lambda Bx}, & 0<x<a \\ \alpha^{+}e^{-\lambda Bx}+\alpha^{-}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}e^{-\lambda B(2a-x)}, & a<x<+\infty \end{cases} \tag{2.1}$$

where $\alpha^{+}=\frac{1}{2}\left(\frac{1}{\alpha}+\alpha\right)$ and $\alpha^{-}=\frac{1}{2}\left(\frac{1}{\alpha}-\alpha\right)$.

Let $Y(x,\lambda)$ be a solution of matrice equation (1.1) satisfying the initial condition $Y(0,\lambda)=I$ and discontinuity conditions (1.3).

Let's show that the function $Y(x,\lambda)$ as

$$Y(x,\lambda)=Y_0(x,\lambda)+\int_x^{\infty}K(x,t)e^{-\lambda Bt}dt \tag{2.2}$$

where $K(x,t)$ is a seconder matrice function. It is obvious that the solution $Y(x,\lambda)$ satisfies the integral equation

$$Y(x,\lambda)=Y_0(x,\lambda)+\int_x^{\infty}Y_0(x,\lambda)Y_0^{-1}(t,\lambda)B\Omega(t)Y(t,\lambda)dt \tag{2.3}$$

to satisfy the of function $Y(x,\lambda)$ given as the type of (2.2), it is necessary that the equality

$$\int_x^{\infty}K(x,t)e^{-\lambda Bt}dt=\int_x^{\infty}Y_0(x,\lambda)Y_0^{-1}(t,\lambda)B\Omega(t)\left[Y_0(t,\lambda)+\int_t^{\infty}K(x,t)e^{-\lambda Bt}ds\right]dt \tag{2.4}$$

must be satisfied. On the constrary that if the matrice function $K(x,t)$ satisfies the equality (2.4), the function $Y(x,\lambda)$ satisfies the equation (2.2).

We shall transform the right hand side of the equality (2.4) such that it will similar to left hand side of this equality. First, let's assume the following expressions,

$$K_{\pm}(x,t)=\frac{1}{2}\left[K(x,t)\pm BK(x,t)B\right]$$

It is seen clearly from the expressions of the matrice functions $K_{+}(x,t)$ and $K_{-}(x,t)$ that these functions have the properties;

$$K(x,t) = K_+(x,t) - K_-(x,t)$$

$$BK_+(x,t) = \frac{1}{2}\left[BK(x,t) - K(x,t)B\right] = -K_+(x,t)B$$

$$BK_-(x,t) = \frac{1}{2}\left[BK(x,t) + K(x,t)B\right] = K_-(x,t)B$$

If we case the equality

$$Y_0(x,\lambda)Y_0^{-1}(t,\lambda) = \begin{cases} e^{-\lambda B(x-t)}, & 0<t<x<a \\ \alpha^+ e^{-\lambda B(x-t)} + \alpha^- \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} e^{-\lambda B(2a-x-t)}, & t<a<x \\ e^{-\lambda B(x-t)}, & a<t<x \end{cases}$$

and the expression of the functions $K_\pm(x,t)$ and if we transform right hand side of the equality (2.4) such that it will similar to expression in left hand side, we get the following system of integral equations for the matrice function $K_+(x,t)$ and $K_-(x,t)$:

I. for $x<t<a$,

$$K_+(x,t) = -\frac{1}{2}B\Omega\left(\frac{x+t}{2}\right) - \int_x^{(x+t)/2} B\Omega(\xi)K_-(\xi,t+x-\xi)d\xi$$

$$K_-(x,t) = -\int_x^{(x+t)/2} B\Omega(\xi)K_+(\xi,t+x-\xi)d\xi$$

II. for $x>a$, $2a-x<t<x$,

$$K_+(x,t) = -\alpha^+\int_a^{(x+t)/2} B\Omega(\xi)K_+(\xi,t+x-\xi)d\xi - \int_a^{(x+t)/2} B\Omega(\xi)K_-(\xi,t+x-\xi)d\xi$$

$$K_-(x,t) = -\alpha^+\int_a^{(x+t)/2} B\Omega(\xi)K_+(\xi,t+x-\xi)d\xi - \int_a^{(x+t)/2} B\Omega(\xi)K_+(\xi,t+x-\xi)d\xi$$

III. for $x>a$, $x<t<3x-a$,

$$K_+(x,t) = -\frac{\alpha^+}{2}B\Omega\left(\frac{x+t}{2}\right) - \alpha^+\int_a^{(x+t)/2} B\Omega(\xi)K_+(\xi,t+x-\xi)d\xi$$

$$-\alpha^-\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\int_a^{(t-x+2a)/2} B\Omega(\xi)K_-(\xi,t+2a-x-\xi)d\xi - \int_a^{(x+t)/2} B\Omega(\xi)K_-(\xi,t+x-\xi)d\xi$$

$$K_-(x,t)=\alpha^+\int_a^{(x+t)/2}B\Omega(\xi)K_+(\xi,t+x-\xi)d\xi-\int_a^{(x+t)/2}B\Omega(\xi)K_+(\xi,t+x-\xi)d\xi$$

$$-\alpha^-\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\int_a^{(t-x+2a)/2}B\Omega(\xi)K_-(\xi,t+2a-x-\xi)d\xi$$

IV. for $x>a$, $3x-a<t<+\infty$,

$$K_+(x,t)=-\frac{\alpha^+}{2}B\Omega\left(\frac{x+t}{2}\right)-\alpha^+\int_a^{(x+t)/2}B\Omega(\xi)K_+(\xi,t+x-\xi)d\xi$$

$$-\alpha^-\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\int_a^{(t-x+2a)/2}B\Omega(\xi)K_-(\xi,t+2a-x-\xi)d\xi-\int_a^{(x+t)/2}B\Omega(\xi)K_-(\xi,t+x-\xi)d\xi$$

$$K_-(x,t)=-\frac{\alpha^-}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}B\Omega\left(\frac{t-x+2a}{2}\right)-\alpha^-\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\int_a^{(t-x+2a)/2}B\Omega(\xi)K_-(\xi,t+2a-x-\xi)d\xi$$

$$-\int_a^{(x+t)/2}B\Omega(\xi)K_+(\xi,t+x-\xi)d\xi-\alpha^+\int_a^{(x+t)/2}B\Omega(\xi)K_+(\xi,t+x-\xi)d\xi$$

when we write successive approximations for each situations,

I. for $x<t<a$,

$$K_+^0(x,t)=-\frac{1}{2}B\Omega\left(\frac{x+t}{2}\right),\quad K_-^0(x,t)=0$$

$I.$ for $x<t<a$

$$K_+^{(0)}(x,t)=-\frac{1}{2}B\Omega\left(\frac{x+t}{2}\right),\qquad K_-^{(0)}(x,t)=0$$

$$K_+^{(n)}(x,t)=-\int_x^{(x+t)/2}B\Omega(\xi)K_-^{(n-1)}(\xi,t+x-\xi)d\xi,\qquad n=1,2,\ldots$$

$$K_-^{(n)}(x,t)=-\int_x^{(x+t)/2}B\Omega(\xi)K_+^{(n-1)}(\xi,t+x-\xi)d\xi,\qquad n=1,2,\ldots$$

$II.$ for $x>a$, $2a-x<t<x$

$$K_+^{(0)}(x,t)=0,\qquad K_-^{(0)}(x,t)=0$$

$$K_+^{(n)}(x,t)=0,\qquad K_-^{(n)}(x,t)=0,\qquad n=1,2,\ldots$$

$III.$ for $x>a$, $x<t<3x-2a$

$$K_+^{(0)}(x,t)=-\frac{\alpha^+}{2}B\Omega\left(\frac{x+t}{2}\right),\qquad K_-^{(0)}(x,t)=0$$

$$K_+^{(n)}(x,t) = -\alpha^+ \int_a^{(x+t)/2} B\Omega(\xi) K_+^{(n-1)}(\xi, t+x-\xi) d\xi -$$

$$- \int_a^{(x+t)/2} B\Omega(\xi) K_-^{(n-1)}(\xi, t+x-\xi) d\xi$$

$$K_-^{(n)}(x,t) = -\alpha^+ \int_a^{(x+t)/2} B\Omega(\xi) K_+^{(n-1)}(\xi, t+x-\xi) d\xi -$$

$$-\alpha^- \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \int_a^{(t-x+2a)/2} B\Omega(\xi) K_-^{(n-1)}(\xi, t+2a-x-\xi) d\xi -$$

$$- \int_a^{(x+t)/2} B\Omega(\xi) K_+^{(n-1)}(\xi, t+x-\xi) d\xi$$

IV. for $x > a,\ 3x - 2a < t < +\infty$

$$K_+^{(0)}(x,t) = -\frac{\alpha^+}{2} B\Omega\left(\frac{x+t}{2}\right)$$

$$K_-^{(0)}(x,t) = -\frac{\alpha^-}{2} B\Omega\left(\frac{t-x+2a}{2}\right)$$

$$K_+^{(n)}(x,t) = -\alpha^+ \int_a^{(x+t)/2} B\Omega(\xi) K_+^{(n-1)}(\xi, t+x-\xi) d\xi -$$

$$-\alpha^- \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \int_a^{(t-x+2a)/2} B\Omega(\xi) K_-^{(n-1)}(\xi, t+2a-x-\xi) d\xi -$$

$$- \int_a^{(x+t)/2} B\Omega(\xi) K_-^{(n-1)}(\xi, t+x-\xi) d\xi$$

$$K_-^{(n)}(x,t) = -\alpha^+ \int_a^{(x+t)/2} B\Omega(\xi) K_+^{(n-1)}(\xi, t+x-\xi) d\xi -$$

$$-\alpha^- \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \int_a^{(t-x+2a)/2} B\Omega(\xi) K_-^{(n-1)}(\xi, t+2a-x-\xi) d\xi -$$

$$- \int_a^{(x+t)/2} B\Omega(\xi) K_+^{(n-1)}(\xi, t+x-\xi) d\xi$$

Now, let's obtain that written successive approximations are convergent for the situation I. and also get that the evaluation for the vector function $K(x,t)$:

$x < t < a$

$$K_+^{(0)}(x,t) = -\frac{1}{2} B\Omega\left(\frac{x+t}{2}\right), \qquad K_-^{(0)}(x,t) = 0$$

$$\int_x^\infty \left\|K_+^{(0)}(x,t)\right\|dt = \frac{1}{2}\int_x^\infty \|B\|\left\|\Omega\left(\frac{x+t}{2}\right)\right\|dt = \int_x^\infty \|\Omega(t)\|dt \le \sigma(x)$$

$$K_-^{(1)}(x,t) = \int_x^{(x+t)/2} B\Omega(\xi)K_+^{(0)}(x,t)d\xi = \int_x^{(x+t)/2} B\Omega(\xi)B\Omega\left(\frac{x+t}{2}\right)d\xi$$

$$\int_x^\infty \left\|K_-^{(1)}(x,t)\right\|dt \le \frac{\sigma^2(x)}{2!}$$

Analogously, for $n = k$, the truth of the following inequalities;

$$\int_x^\infty \left\|K_+^{(2k)}(x,t)\right\|dt \le \frac{\sigma^{(2k+1)}(x)}{(2k+1)!}$$

$$\int_x^\infty \left\|K_-^{(2k+1)}(x,t)\right\|dt \le \frac{\sigma^{(2k+2)}(x)}{(2k+2)!}$$

is obtained. Let's show that similar inequalities are satisfied for also $n = k+1$:

Since

$$K_+^{(2k+2)}(x,t) = -\int_x^{(x+t)/2} B\Omega(\xi)K_-^{(2k+1)}(\xi, t+x-\xi)d\xi,$$

$$\int_x^\infty \left\|K_+^{(2k+2)}(x,t)\right\|dt \le \int_x^\infty \int_x^{(x+t)/2} \|\Omega(\xi)\|\left\|K_-^{(2k+1)}(\xi, t+x-\xi)\right\|d\xi dt$$

$$= \int_x^\infty \int_{2\xi-x}^\infty \|\Omega(\xi)\|\left\|K_-^{(2k+1)}(\xi, t+x-\xi)\right\|dt d\xi$$

$$= \int_x^\infty \|\Omega(\xi)\|\int_\xi^\infty \left\|K_-^{(2k+1)}(\xi,\tau)\right\|d\tau d\xi$$

$$= -\frac{1}{(2k+2)!}\int_x^\infty \|\Omega(\xi)\|\sigma^{(2k+2)}(\xi)d\xi = \frac{\sigma^{(2k+3)}(x)}{(2k+3)!}$$

Similarly,

$$\int_x^\infty \left\|K_-^{(2k+3)}(x,t)\right\|dt \le \frac{\sigma^{(2k+4)}(x)}{(2k+4)!}$$

We get the following evaluations that series $\sum_{n=0}^\infty \int_x^\infty \left\|K_\pm^{(n)}(x,t)\right\|dt$ are absolutely and uniformly convergent with respect to x over $[0,\pi]$. Then if we consider that $K(x,t) = K_+(x,t) + K_-(x,t)$ then we get

$$\int_x^\infty \|K(x,t)\| dt \le e^{\sigma(x)} - 1.$$

Let's assume that $\Omega(x)$ function is differentiable. In this case when the expression of the function $Y(x,\lambda)$ substitute in the equation (1.1):

$$B\frac{dy}{dx} + y = \lambda y$$

$$B\frac{d}{dx}\left(Y_0(x,\lambda) + \int_x^\infty K(x,t)e^{-\lambda Bt}dt \right) + \Omega(x)\left(Y_0(x,\lambda) + \int_x^\infty K(x,t)e^{-\lambda Bt}dt \right)$$

$$= \lambda\left(Y_0(x,\lambda) + \int_x^\infty K(x,t)e^{-\lambda Bt}dt \right)$$

$$BY_0'(x,\lambda) + B\frac{d}{dx}\left(\int_x^\infty K(x,t)e^{-\lambda Bt}dt \right) + \Omega(x)Y_0(x,\lambda) + \Omega(x)\int_x^\infty K(x,t)e^{-\lambda Bt}dt$$

$$= \lambda Y_0(x,\lambda) + \lambda\int_x^\infty K(x,t)e^{-\lambda Bt}dt$$

here since $BY_0'(x,\lambda) = \lambda Y_0(x,\lambda)$,

$$B\frac{d}{dx}\left(\int_x^{2a-x} K(x,t)e^{-\lambda Bt}dt + \int_{2a-x}^{3x-2a} K(x,t)e^{-\lambda Bt}dt + \int_{3x-2a}^\infty K(x,t)e^{-\lambda Bt}dt \right) +$$

$$+\Omega(x)Y_0(x,\lambda) + \Omega(x)\int_x^\infty K(x,t)e^{-\lambda Bt}dt = \lambda\int_x^\infty K(x,t)e^{-\lambda Bt}dt$$

$$B\Big[-K(x,2a-x-0)e^{-\lambda B(2a-x)} - K(x,x)e^{-\lambda Bx} + 3K(x,3x-2a-0)e^{-\lambda B(3x-2a)} +$$

$$+K(x,2a-x+0)e^{-\lambda B(2a-x)} - 3K(x,3x-2a+0)e^{-\lambda B(3x-2a)}\Big] + B\int_x^\infty \frac{\partial K(x,t)}{\partial x}e^{-\lambda Bt}dt +$$

$$+\Omega(x)Y_0(x,\lambda) + \Omega(x)\int_x^\infty K(x,t)e^{-\lambda Bt}dt = \lambda\int_x^\infty K(x,t)e^{-\lambda Bt}dt$$

is obtained.

When partial integral is applied one time in the integral which is right hand side of last equality

$$B\big[K(x,2a-x+0) - K(x,2a-x-0)\big]e^{-\lambda B(2a-x)} - BK(x,x)e^{-\lambda Bx} -$$
$$-3B\big[K(x,3x-2a+0) - K(x,3x-2a-0)\big]e^{-\lambda B(3x-2a)} + \Omega(x)Y_0(x,\lambda)$$
$$+\int_x^\infty \left[B\frac{\partial K(x,t)}{\partial x} + \Omega(x)K(x,t) \right]e^{-\lambda Bt}dt =$$

$$= [K(x,2a-x+0)-K(x,2a-x-0)]Be^{-\lambda B(2a-x)} - K(x,x)Be^{-\lambda Bx} -$$
$$-3[K(x,3x-2a+0)-K(x,3x-2a-0)]Be^{-\lambda B(3x-2a)} - \int_x^{\infty} \frac{\partial K(x,t)}{\partial x} Be^{-\lambda Bt}\,dt$$

from this last equality,

1) $B\dfrac{\partial K(x,t)}{\partial x} + \Omega(x)K(x,t) = -\dfrac{\partial K(x,t)}{\partial t}B$

2) for $0 < x < a$

$-BK(x,x) + \Omega(x)K(x,x) = -K(x,x)B$

3) for $x < a$

$$B[K(x,2a-x+0)-K(x,2a-x-0)] + \Omega(x) =$$
$$= -[K(x,2a-x+0)-K(x,2a-x-0)]B$$

4) for $x > a$

$$-BK(x,x) + \alpha^{+}\Omega(x) = -K(x,x)B,$$
$$B[K(x,2a-x+0)-K(x,2a-x-0)] + \alpha^{-}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\Omega(x) = \qquad (2.5)$$
$$= -[K(x,2a-x+0)-K(x,2a-x-0)]B,$$
$$3B[K(x,3x-2a+0)-K(x,3x-2a-0)] = [K(x,3x-2a+0)-K(x,3x-2a-0)]B$$

Thus we have been proved following theorem:

Theorem: Let's say that $\int_x^{\infty} \|\Omega(x)\| dx < +\infty$ then each solution $Y(x,\lambda)$ satisfying conditions (1.2)-(1.3) of the equation (1.1) has the expression (2.2) and moreover $\int_x^{\infty} \|K(x,t)\| dt \le e^{\sigma(x)} - 1$ where $\sigma(x) = \int_x^{\infty} \|\Omega(t)\| dt$.

If the function $\sigma(x)$ is differentiable then the function $K(x,t)$ satisfies the conditions of (2.5).

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